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# HOW SMALL THE NUMBER OF TEST ITEMS CAN BE FOR THE BASIS OF ESTIMATING THE OPERATING CHARACTERISTICS OF THE DISCRETE RESPONSES TO UNKNOWN TEST ITEMS

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# HOW SMALL THE NUMBER OF TEST ITEMS CAN BE FOR THE BASIS OF ESTIMATING THE OPERATING CHARACTERISTICS OF THE DISCRETE RESPONSES TO UNKNOWN TEST ITEMS


## ABSTRACT

The methods and approaches for estimating the operating characteristics of the discrete item responses without assuming any mathematical form have been developed and expanded. It has been made possible that, even if the test information function of a given test is not constant for the interval of ability of our interest, we use it as our Old Test. Our original Old Test consists of thirty-five test items with three item score categories each. Recently, we experimented upon the Simple Sum Procedure of the Conditional P.D.F. Approach which is combined with the Normal Approach Method, using each of three subtests of the original Old Test, and the results turned out to be successful. In the present study, we experiment on the same combination of approach and method by using several subtests of the original Old Test, which contain fifteen or less test items each, to find out if each of them can still be used as the Old Test, which maintains the accuracy of estimation reasonably high.





The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee. Those who worked in the laboratory and helped the authors in various ways for this research include Charles McCarter, C. I. Bonnie Chen, Zailan Mohamed, Shiao-Yung Chen and Pat Palmer.

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# I Introduction

In previous studies (Samejima, RR-80-4, RR-81-2), three subtests of the original Old Test, i.e., Subtests 1, 2 and 3, have each been used as the Old Test, in place of the original one, in the estimation of the item characteristic functions of ten unknown, binary test items. The main features of these studies are: 1) the number of test items in each Old Test is much less than that of the original Old Test and, as the result, the amount of test information is less than that of the original Old Test for any fixed value of ability $\theta$ ; 2) the test information function of each Old Test is not constant for the interval of $\theta$ of our interest, making it necessary to transform ability $\theta$ to its strictly increasing function $\tau$ , for which our Old Test has a constant test information function; and 3) in so doing, the method of moments for fitting polynomials, which also is the least squares solution (Samejima and Livingston, RR-79-2), is effectively used.

In these studies, the combination of the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method was consistently used. Our calibration data are based upon the five hundred hypothetical examinees whose ability levels are one hundred equally spaced positions starting from -2.475 and ending with 2.475 , with five examinees sharing each position. The results turned out to be quite successful.

In the present study, we shall investigate the possibility of further reducing the number of test items in the Old Test. In other words, we shall find out how robust our methods can be over the reduction in the amount of test information of the Old Test. Throughout the present study, we shall use the same combination of a method and an approach and the same group of hypothetical examinees as in the previous studies.

## II Six Subtests of the Original Old Test

Let $g$ denote an item of the original Old Test, and $x_g$ $(=0,1,...,m_g)$ be the graded item score of item $g$. The operating characteristic of the graded item score $x_g$ is denoted by $P_{x_g}(\theta)$, and follows the normal ogive model such that

$$(2.2) \qquad P_{x_g}(\theta) = (2\pi)^{-1/2} \int_{a_g(\theta-b_{x_g+1})}^{a_g(\theta-b_{x_g})} e^{-u^2/2}\, du \quad ,$$

where $a_g$ $(>0)$ is the item discrimination parameter, and $b_{x_g}$ is the item response difficulty parameter which satisfies

$$(2.2) \qquad -\infty = b_0 < b_1 < b_2 < \ldots\ldots < b_{m_g} < b_{(m_g+1)} = \infty \quad .$$

With our original Old Test, $m_g = 2$ for each item. Table 2-1 presents the item discrimination parameter, $a_g$, and the two item response difficulty parameters, $b_1$ and $b_2$, for each of the thirty-five test items.

There are six subtests of the original Old Test, each of which we used as the Old Test in the present study. They are called Subtests 4, 5, 6, 7, 8 and 9, respectively. The numbers of test items included by these six subtests are 15, 15, 11, 9, 7 and 5, respectively. These test items are marked with crosses in Table 2-1, indicating which item belongs to each subtest. We notice that Subtest 4 is a subtest of Subtest 2, as well as of the original Old Test, just as Subtest 3 is a subtest of Subtest 1 (Samejima, RR-80-4, RR-81-2).

The importance of the square root of the test information function, $I(\theta)$, of the Old Test in the estimation of the operating characteristics of the discrete item responses has previously been demonstrated (Samejima, RR-80-2, RR-80-4, RR-81-2). Among other reasons, it has an important role in transforming ability $\theta$ to $\tau$, when it is not

TABLE 2-1

Item Discrimination Parameter, $a_g$ , and Item Response Difficulty Parameters, $b_{x_g}$ , for $x_g = 1$ and $x_g = 2$ , for Each of the Thirty-five Test Items of the Original Old Test. Items Included by Subtests 4, 5, 6, 7, 8 and 9 Are Marked By Crosses, Respectively.

| Item g | $a_g$ | $b_1$ | $b_2$ | Subtest 4 | Subtest 5 | Subtest 6 | Subtest 7 | Subtest 8 | Subtest 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.8 | -4.75 | -3.75 | | | | | | |
| 2 | 1.9 | -4.50 | -3.50 | | | | x | | |
| 3 | 2.0 | -4.25 | -3.25 | | | x | | x | |
| 4 | 1.5 | -4.00 | -3.00 | | x | | | | x |
| 5 | 1.6 | -3.75 | -2.75 | | | | | | |
| 6 | 1.4 | -3.50 | -2.50 | x | x | x | x | | |
| 7 | 1.9 | -3.00 | -2.00 | x | | | | | |
| 8 | 1.8 | -3.00 | -2.00 | x | x | | | x | |
| 9 | 1.6 | -2.75 | -1.75 | x | | x | | | |
| 10 | 2.0 | -2.50 | -1.50 | x | x | | x | | |
| 11 | 1.5 | -2.25 | -1.25 | x | | | | | x |
| 12 | 1.7 | -2.00 | -1.00 | x | x | x | | | |
| 13 | 1.5 | -1.75 | -0.75 | | | | | x | |
| 14 | 1.4 | -1.50 | -0.50 | | x | | x | | |
| 15 | 2.0 | -1.25 | -0.25 | | | x | | | |
| 16 | 1.6 | -1.00 | 0.00 | | x | | | | |
| 17 | 1.8 | -0.75 | 0.25 | | | | | | |
| 18 | 1.7 | -0.50 | 0.50 | | x | x | x | x | x |
| 19 | 1.9 | -0.25 | 0.75 | | | | | | |
| 20 | 1.7 | 0.00 | 1.00 | | x | | | | |
| 21 | 1.5 | 0.25 | 1.25 | | | x | | | |
| 22 | 1.8 | 0.50 | 1.50 | | x | | x | | |
| 23 | 1.4 | 0.75 | 1.75 | x | | | | x | |
| 24 | 1.9 | 1.00 | 2.00 | x | x | x | | | |
| 25 | 2.0 | 1.25 | 2.25 | x | | | | | x |
| 26 | 1.6 | 1.50 | 2.50 | x | x | | x | | |
| 27 | 1.7 | 1.75 | 2.75 | x | | x | | | |
| 28 | 1.4 | 2.00 | 3.00 | x | x | | | x | |
| 29 | 1.9 | 2.25 | 3.25 | x | | | | | |
| 30 | 1.6 | 2.50 | 3.50 | x | x | x | x | | |
| 31 | 1.5 | 2.75 | 3.75 | | | | | | |
| 32 | 1.7 | 3.00 | 4.00 | | x | | | | x |
| 33 | 1.8 | 3.25 | 4.25 | | | x | | x | |
| 34 | 2.0 | 3.50 | 4.50 | | | | x | | |
| 35 | 1.4 | 3.75 | 4.75 | | | | | | |

constant for the interval of $\theta$ of our interest. Figure 2-1 presents the square root of the test information function of each of the six subtests by solid curves.

Polynomials of degree 3, 4, 5, 6 and 7 were fitted to the square root of the test information of each subtest by the method of moments. These polynomials, $\sum_{k=0}^{m} \alpha_k \theta^k$ (m=3,4,5,6,7), which were obtained by using the interval of $\theta$, [-4.0, 4.0], are plotted in Figure 2-1 by dots, for each of the six subtests. Out of these five polynomials for each subtest, that of degree 7 was selected as the approximation to the square root of the test information function and used in the process of the transformation of $\theta$ to $\tau$. Table 2-2 presents the coefficients $\alpha_k$ (k=0,1,2,3,4,5,6,7) of this polynomial and, in Appendix as Table A-1, we find those for the other four polynomials, for each subtest. It has been pointed out (Samejima and Livingston, RR-79-2) that the selection of an appropriate interval of $\theta$ is very important in fitting a polynomial for a specific function. From Figure 2-1, we can see that for each subtest the polynomial of degree 7 fits well to the square root of the test information function. We also experimented with the interval, [-3.0, 3.0], for Subtest 4, and the results turned out to be less than satisfactory. These results are shown in Appendix, as Figure A-1, and their coefficients are presented in Table A-2.

It has been shown (Samejima, RR-80-2) that the transformation of ability $\theta$ to $\tau$, which provides us with a constant amount of test information, $I^*(\tau) = C^2$, for our Old Test, can be obtained as another polynomial of degree $m+1$, such that

$$(2.3) \qquad \tau = \sum_{k=0}^{m+1} \alpha_k^* \theta^k ,$$

where

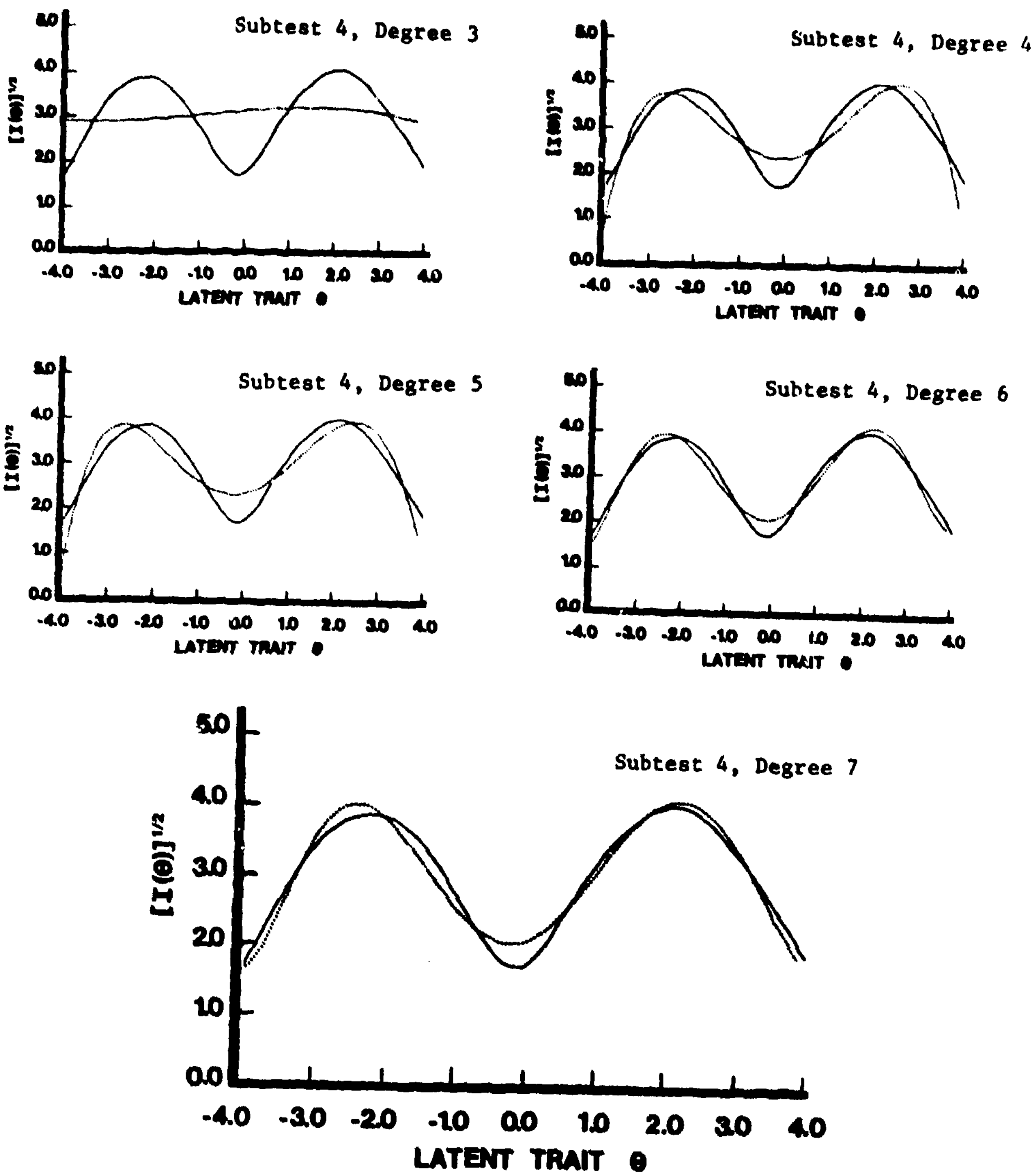
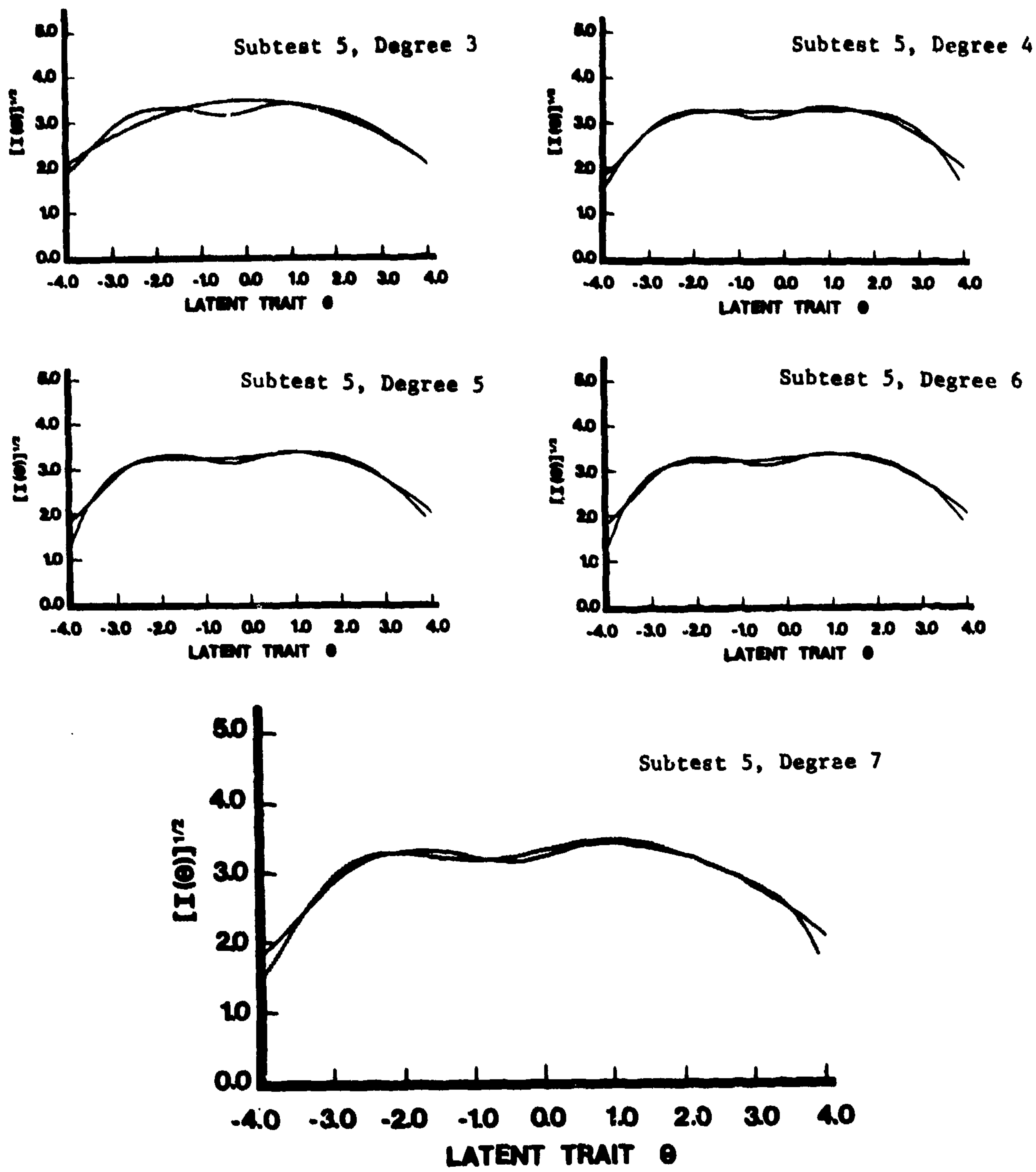



FIGURE 2-1

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Curve) of Each of the Six Subtests and Its Approximation by a Polynomial (Dotted Curve), Which was Fitted by the Method of Moments with $[-4.0, 4.0]$ as the Interval of $\theta$. The Degrees of the Five Polynomials are 3, 4, 5, 6 and 7, Respectively.

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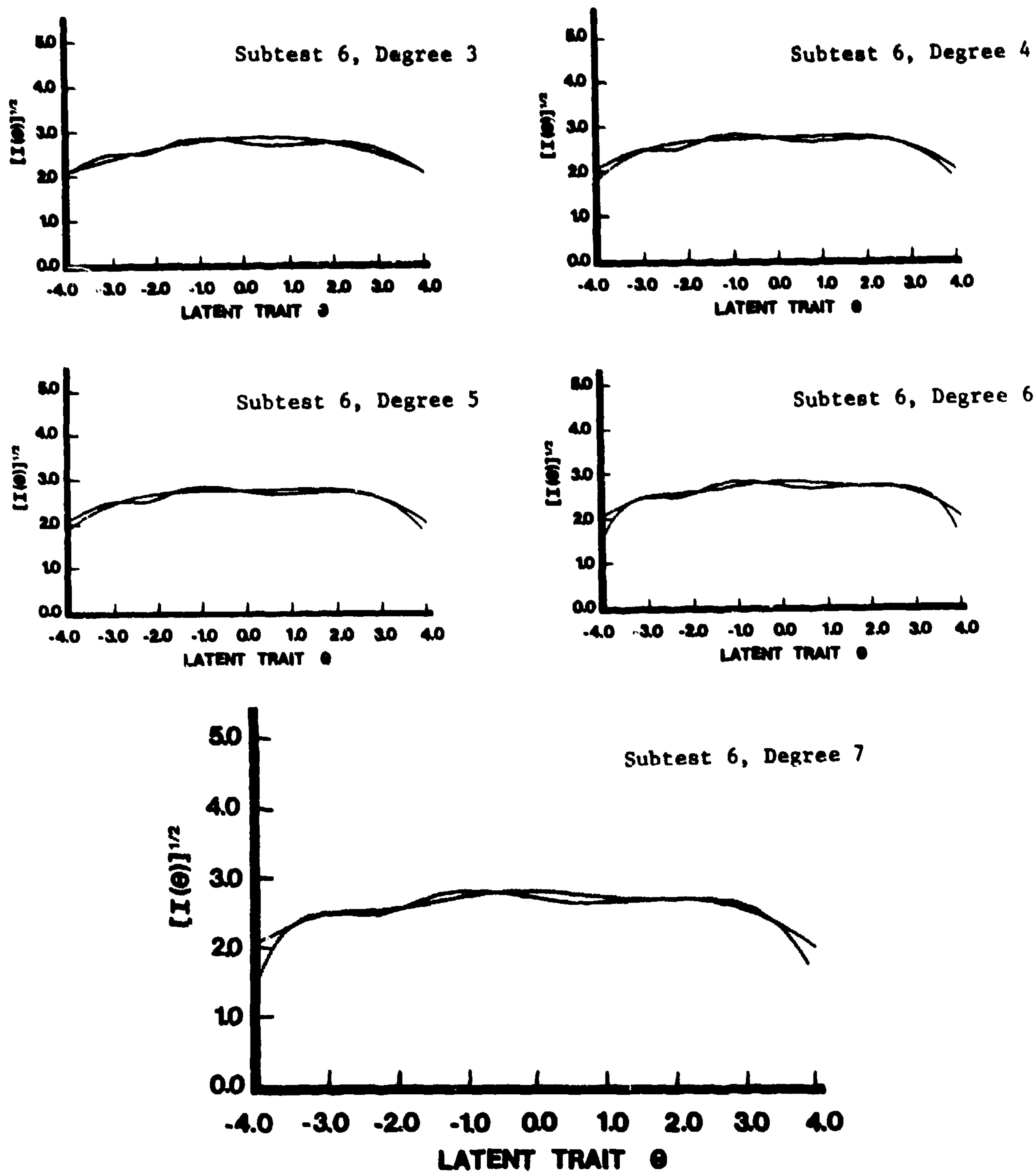


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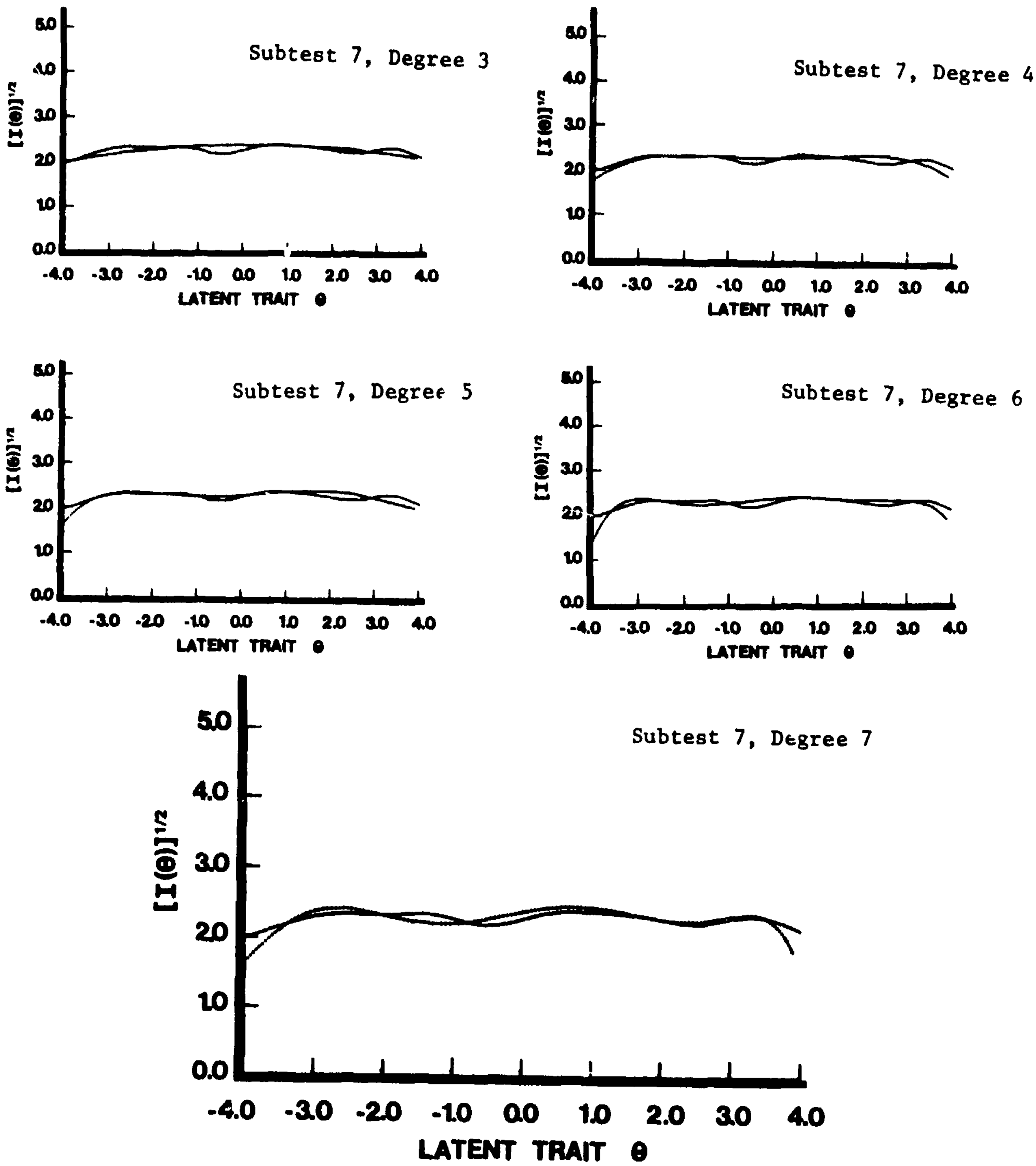


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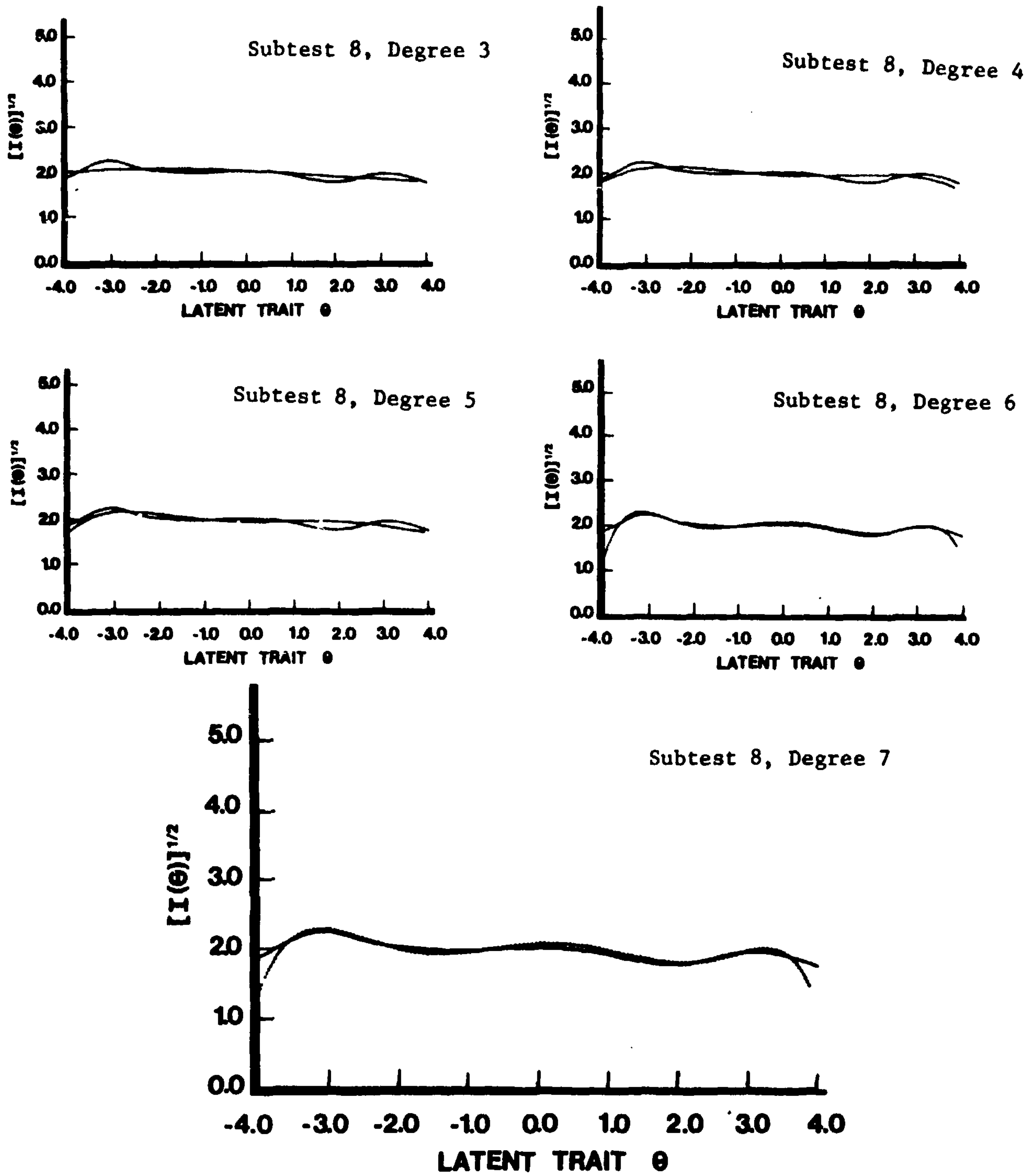


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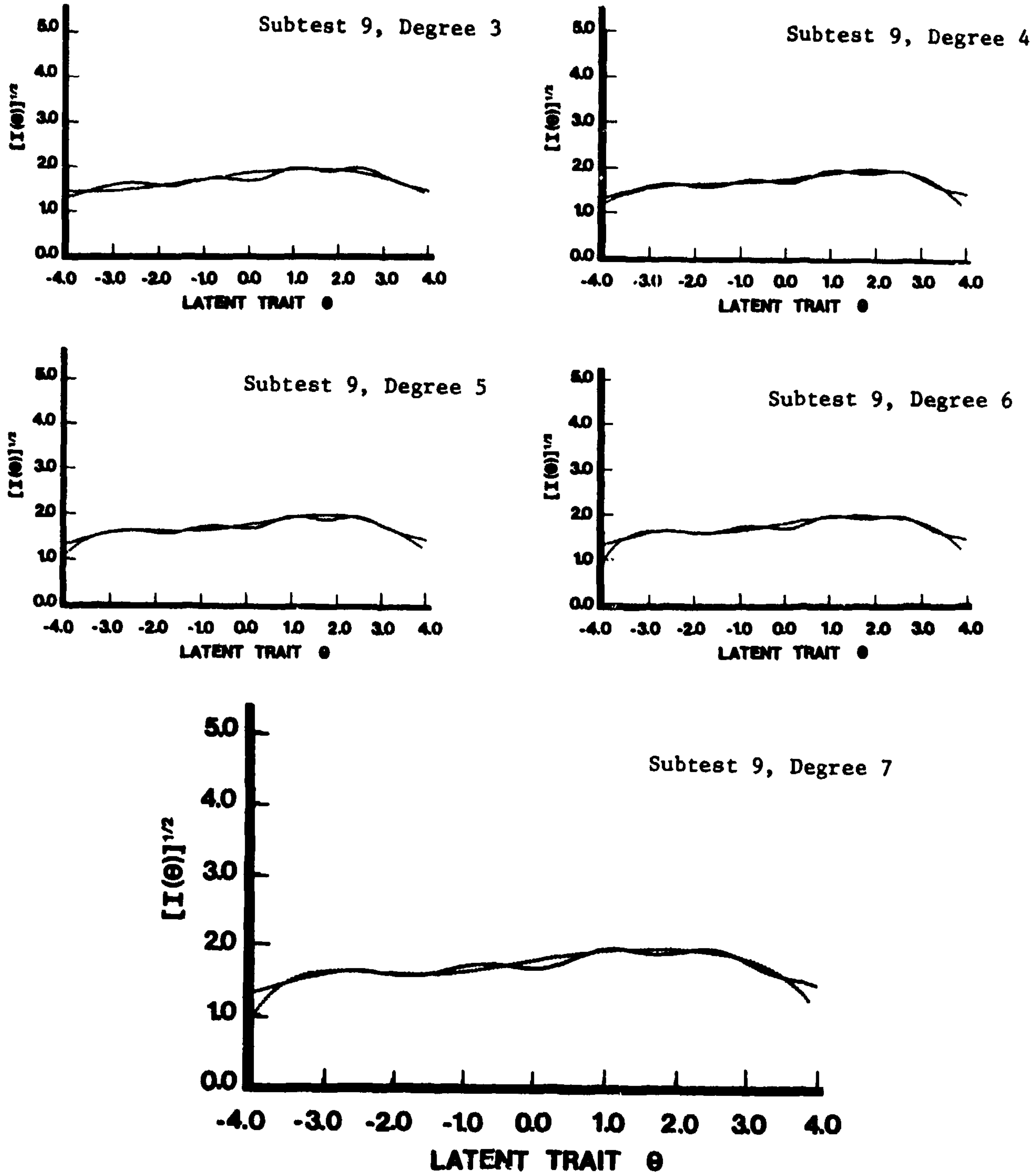



FIGURE 2-1 (Continued)

TABLE 2-2

Coefficients of the Polynomial of Degree 7 Obtained by the Method of Moments Using the Interval of $\theta$ , (-4.0, 4.0), to Approximate the Square Root of the Test Information Function for Each of the Six Subtests.

| k | $\alpha_k$ | | |
|---|---|---|---|
| | Subtest 4 | Subtest 5 | Subtest 6 |
| 0 | 0.20653741D+01 | 0.33028985D+01 | 0.28244081D+01 |
| 1 | 0.30418478D+00 | 0.23005688D+00 | -0.38865645D-01 |
| 2 | 0.83359597D+00 | 0.68038274D-02 | -0.89251467D-01 |
| 3 | -0.99045455D-01 | -0.10158474D+00 | 0.31726461D-01 |
| 4 | -0.10345640D+00 | -0.45492312D-02 | 0.14561530D-01 |
| 5 | 0.10055738D-01 | 0.11649033D-01 | -0.41758069D-02 |
| 6 | 0.31140746D-02 | -0.17744225D-03 | -0.86600132D-03 |
| 7 | -0.31461009D-03 | -0.38798596D-03 | 0.14748755D-03 |

| k | $\alpha_k$ | | |
|---|---|---|---|
| | Subtest 7 | Subtest 8 | Subtest 9 |
| 0 | 0.23698838D+01 | 0.20794533D+01 | 0.18031475D+01 |
| 1 | 0.20090568D+00 | 0.44515686D-01 | 0.17911118D+00 |
| 2 | -0.67057488D-01 | -0.13624656D+00 | -0.17773403D-01 |
| 3 | -0.89954850D-01 | -0.38893432D-01 | -0.28202923D-01 |
| 4 | 0.14720463D-01 | 0.28958024D-01 | 0.46765970D-02 |
| 5 | 0.10798462D-01 | 0.43578763D-02 | 0.15433890D-02 |
| 6 | -0.85028384D-03 | -0.14683524D-02 | -0.40858826D-03 |
| 7 | -0.37494191D-03 | -0.13598905D-03 | -0.27032925D-04 |

$$\alpha_k^* \begin{cases} = d & \text{for } k = 0 \\ = (Ck)^{-1}\ \alpha_{k-1} & \text{for } k = 1,2,\ldots\ldots,m,m+1\ , \end{cases} \tag{2.4}$$

where $d$ is an arbitrarily set constant for adjusting the origin of $\tau$ . For our purpose, we used $C = 3.5$ , and $d = 0$ . Table 2-3 presents the resultant nine coefficients, $\alpha_k^*$ $(k=0,1,2,3,4,5,6,7,8)$ , for each of the six subtests. Figure 2-2 presents the functional relationship between $\theta$ and $\tau$ , for each subtest. As is expected from the square root of the test information function of each subtest, which is shown in Figure 2-1, the transformation of $\theta$ to $\tau$ is close to linear, except for Subtest 4. The interval of $\tau$ corresponding to the interval of $\theta$ , $[-4.0, 4.0]$ , is $[-3.36106, 3.57935]$ for Subtest 4, $[-3.39905, 3.46366]$ for Subtest 5, $[-2.93479, 2.97885]$ for Subtest 6, $[-2.54949, 2.63541]$ for Subtest 7, $[-2.33633, 2.18084]$ for Subtest 8 and $[-1.82137, 2.08427]$ for Subtest 9 . We notice that, as the number of test items decreases, the interval of $\tau$ is shortened. This comes from the fact that the square root of the test information decreases as the number of test items decreases, and yet we used the same constant, $C = 3.5$ , for each of the six subtests.

Figure 2-3 presents the square root of the test information function, $[I^*(\tau)]^{1/2}$ , which was obtained by the polynomial transformation of $\theta$ to $\tau$ , such that

$$\begin{aligned} [I^*(\tau)]^{1/2} &= [I(\theta)]^{1/2} \frac{d\theta}{d\tau} \\ &= [I(\theta)]^{1/2}\ C\ [\sum_{k=0}^{m} \alpha_k\ \theta^k]^{-1}\ , \end{aligned} \tag{2.5}$$

by dots, together with the horizontal line indicating $C = 3.5$ , for each of the six subtests. We can see in this figure that the actual square root of the test information is very close to the target constant, for most subtests. For Subtest 4 and Subtest 9, however, we can see some noticeable discrepancies between the two.

TABLE 2-3

Coefficients of the Polynomial of Degree 8 Transforming $\theta$ to $\tau$ for Each of the Six Subtests.

| k | $\alpha_k^*$ | | |
|---|---|---|---|
| | Subtest 4 | Subtest 5 | Subtest 6 |
| 0 | 0.00000000D+00 | 0.00000000D+00 | 0.00000000D+00 |
| 1 | 0.59010689D+00 | 0.94368529D+00 | 0.80697374D+00 |
| 2 | 0.43454969D-01 | 0.32865269D-01 | -0.55522350D-02 |
| 3 | 0.79390092D-01 | 0.64798356D-03 | -0.85001397D-02 |
| 4 | -0.70746754D-02 | -0.72560529D-02 | 0.22661758D-02 |
| 5 | -0.59117943D-02 | -0.25995607D-03 | 0.83208743D-03 |
| 6 | 0.47884467D-03 | 0.55471586D-03 | -0.19884795D-03 |
| 7 | 0.12710509D-03 | -0.72425408D-05 | -0.35346993D-04 |
| 8 | -0.11236075D-04 | -0.13856641D-04 | 0.52674125D-05 |

| k | $\alpha_k^*$ | | |
|---|---|---|---|
| | Subtest 7 | Subtest 8 | Subtest 9 |
| 0 | 0.00000000D+00 | 0.00000000D+00 | 0.00000000D+00 |
| 1 | 0.67710966D+00 | 0.59412951D+00 | 0.51518500D+00 |
| 2 | 0.28700811D-01 | 0.63593837D-02 | 0.25587311D-01 |
| 3 | -0.63864274D-02 | -0.12975863D-01 | -0.16927050D-02 |
| 4 | -0.64253464D-02 | -0.27781023D-02 | -0.20144945D-02 |
| 5 | 0.84116931D-03 | 0.16547442D-02 | 0.26723411D-03 |
| 6 | 0.51421248D-03 | 0.20751792D-03 | 0.73494714D-04 |
| 7 | -0.34705463D-04 | -0.59932751D-04 | -0.16677072D-04 |
| 8 | -0.13390782D-04 | -0.48567518D-05 | -0.96546161D-06 |

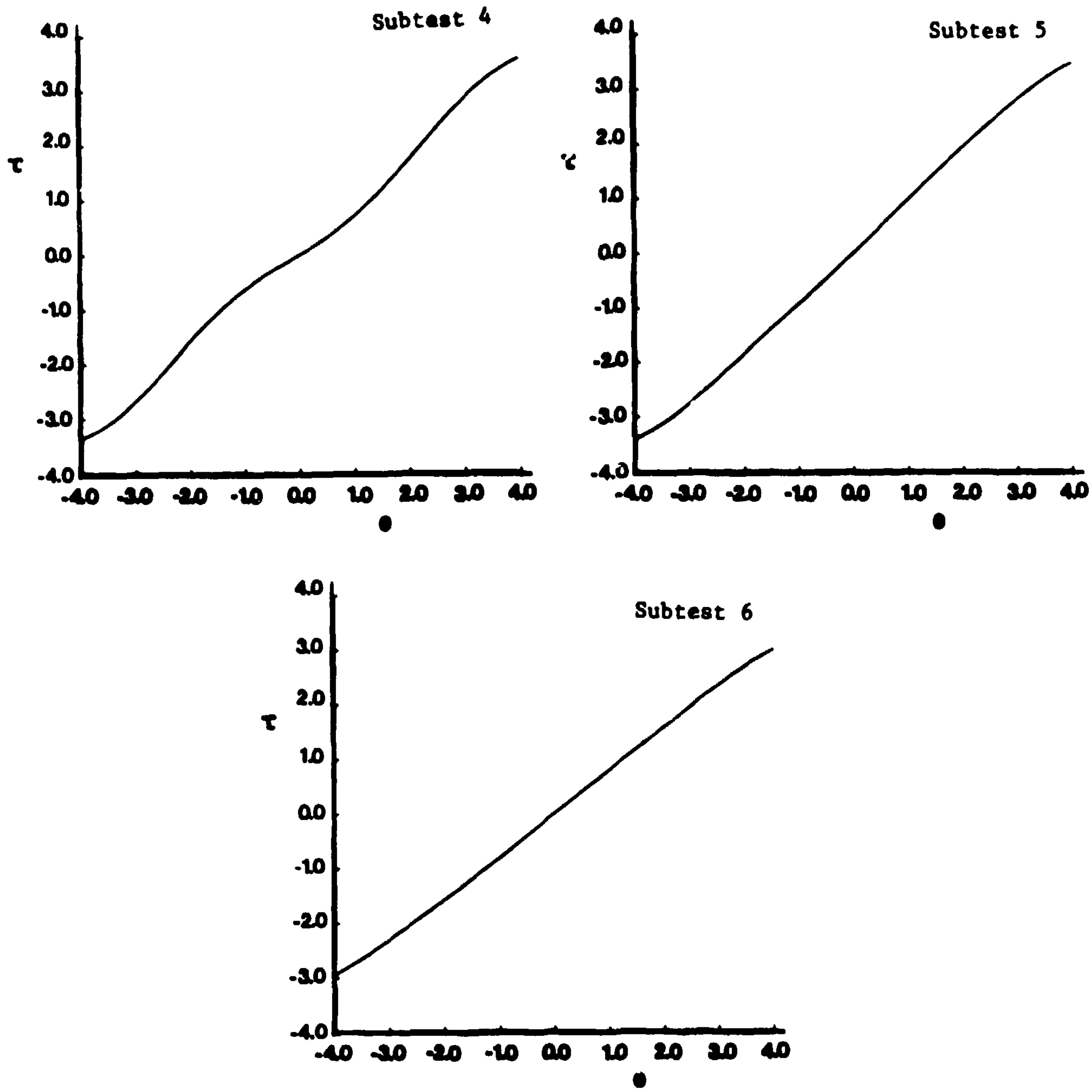



FIGURE 2-2

Functional Relationship Between the Transformed Latent Trait $\tau$ and the Original Latent Trait $\theta$, Based upon the Polynomial of Degree 8, for Each of the Six Subtests.

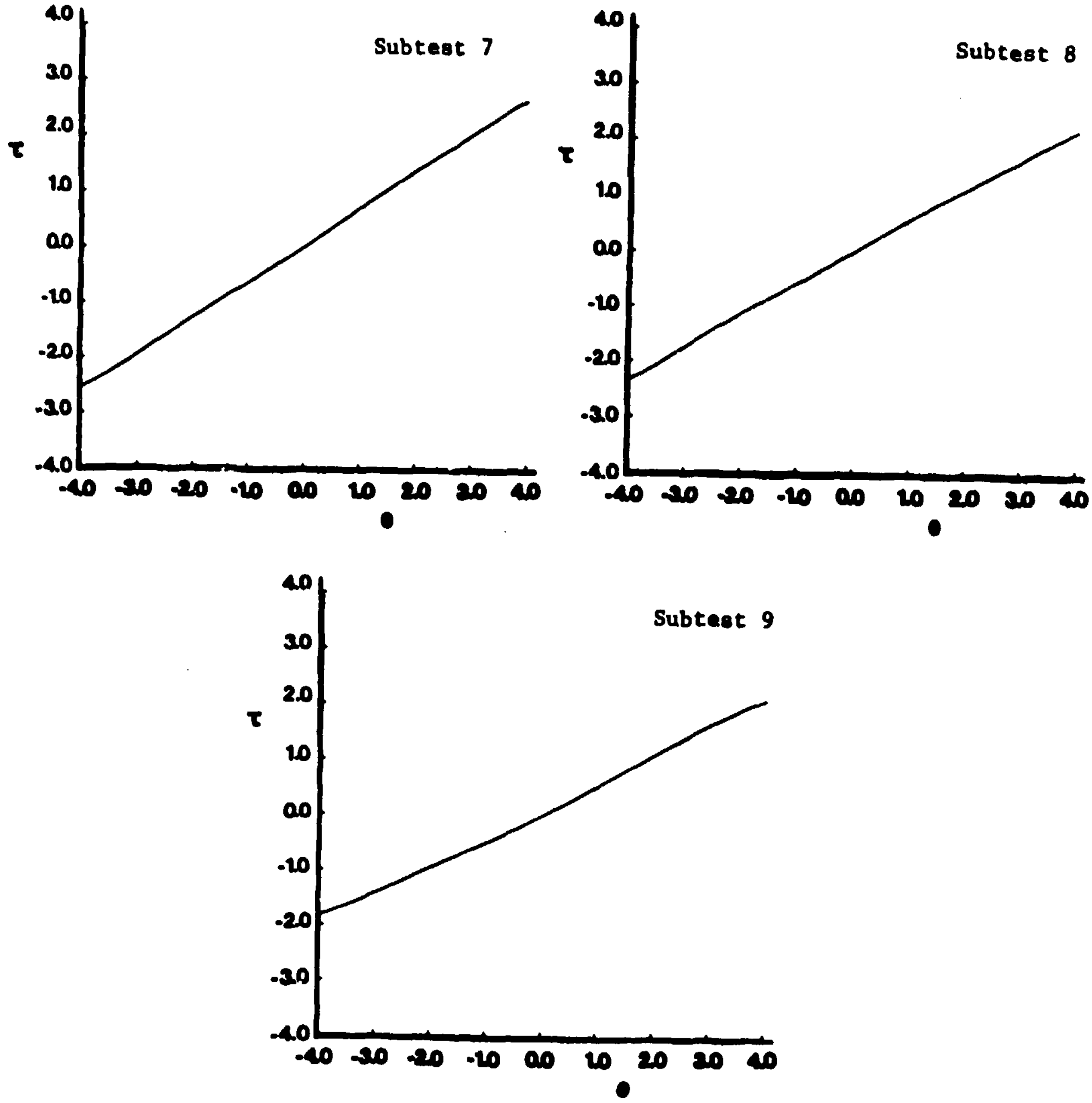
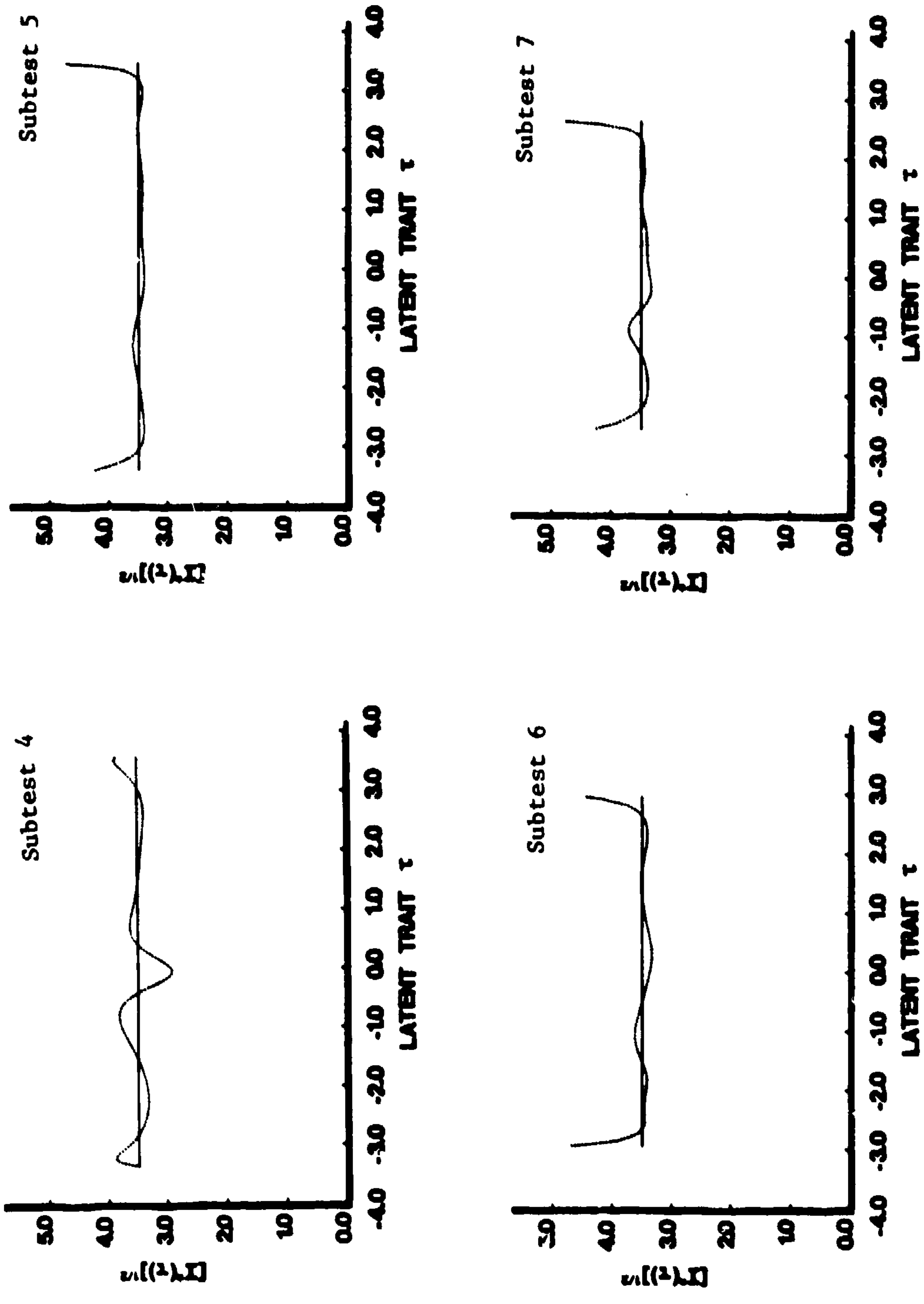



FIGURE 2-2 (Continued)


Subtest 5
Subtest 7
Subtest 4
Subtest 6
LATENT TRAIT τ
-4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0
0.0 1.0 2.0 3.0 4.0 5.0

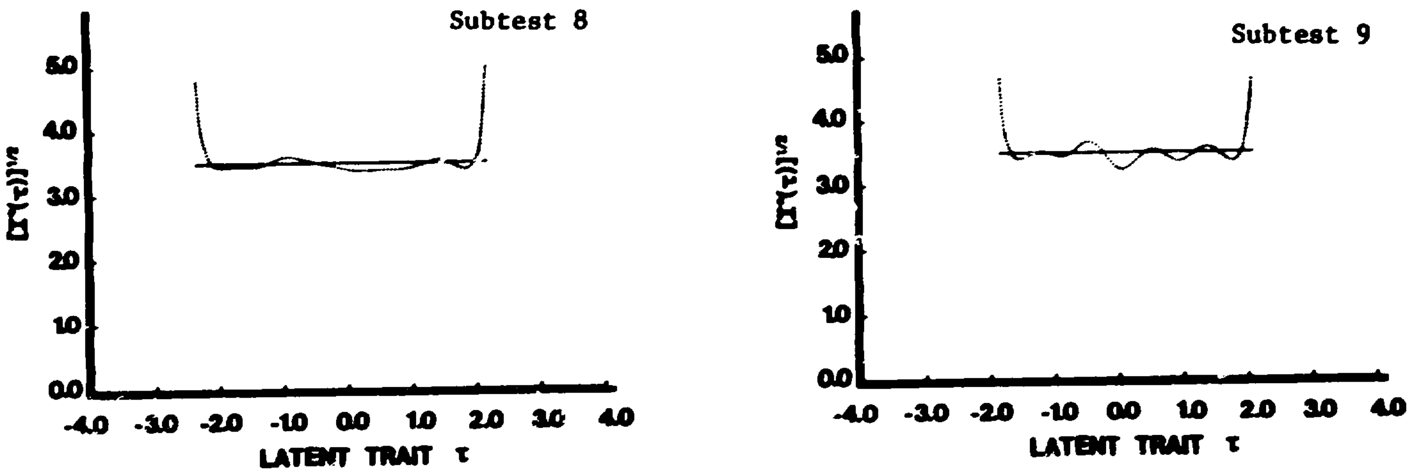



FIGURE 2-3

Square Root of the Test Information Function, $[I^*(\theta)]^{1/2}$, (Dotted Curve), Resultant from the Transformation of $\theta$ to $\tau$ by the Polynomial of Degree 8, in Comparison with the Target Constant Amount of 3.5 (Solid Line), for Each of the Six Subtests.

## III Approximation of the Density Function of $\hat{\tau}$ by a Polynomial Using the Method of Moments

As was mentioned in Chapter 1, in the present study, we used the same five hundred hypothetical examinees as we did in the previous studies (Samejima, RR-77-1, RR-78-1, RR-78-2, RR-78-3, RR-78-4, RR-78-5, RR-78-6, RR-80-2, RR-80-4, RR-81-2). The maximum likelihood estimate, $\hat{\theta}_s$ , for each examinee $s$ , was obtained from his response pattern for each of the six subtests, Subtests 4 through 9, using the basic functions (cf. Samejima, 1969, 1972). As it turned out, for Subtest 9, two examinees, No. 2 and No. 59, obtained negative and positive infinities as their maximum likelihood estimates, respectively. Since there are only two non-finite estimates, we excluded these two examinees from the total group, instead of using some substitute estimates (cf. Samejima, RR-80-3, RR-81-1), and used the remaining 498 examinees for Subtest 9, whereas for all the other five subtests the original group of 500 examinees was used.

By virtue of the transformation-free character of the maximum likelihood estimate (Samejima, 1969), $\hat{\theta}_s$ thus obtained can be transformed to the other maximum likelihood estimate, $\hat{\tau}_s$ , through the same polynomial transformation given by (2.3). Using (2.3) and the coefficients, $\alpha_k^*$ $(k=0,1,2,3,4,5,6,7,8)$ , which are shown in Table 2-3, we transformed $\hat{\theta}_s$ to $\hat{\tau}_s$ for the 500 examinees for each of the Subtests 4 through 8, and for the 498 examinees for Subtest 9.

Table 3-1 presents the first through tenth sample moments about the origin, and those about the midpoint, of the set of $\hat{\tau}_s$ , which is based upon each of the six Old Tests, i.e. Subtests 4 through 9. The midpoint of the set of $\hat{\tau}_s$ 's turned out to be 0.1026 for Subtest 4, 0.0684 for Subtest 5, 0.1934 for Subtest 6, 0.0426 for Subtest 7, -0.2807 for Subtest 8, and 0.1175 for Subtest 9. This midpoint and the first $p$ moments about the midpoint were used in applying the method of moments (Elderton and Johnson, 1969) for fitting a polynomial of degree $p$ , the detailed procedure of which is described in a previous study (Samejima, RR-77-1). Table 3-2 presents the resultant five sets

TABLE 3-1

First Through Tenth Sample Moments of $\hat{\tau}$ about the Origin for Our Hypothetical Examinees, and the Corresponding Sample Moments about the Midpoint, for Each of the Six Subtests. Number of Hypothetical Examinees in Each Sample is Also Shown.

| | Moments About Origin | | | | | |
|---|---|---|---|---|---|---|
| | Subtest 4 | Subtest 5 | Subtest 6 | Subtest 7 | Subtest 8 | Subtest 9 |
| 1 | 0.54117400D-01 | 0.22786000D-01 | -0.20140000D-03 | 0.23163600D-01 | -0.10683600D-01 | 0.53347791D-01 |
| 2 | 0.14361094D 01 | 0.20003610D 01 | 0.14040600D 01 | 0.10375697D 01 | 0.76677091D 00 | 0.64101940D 00 |
| 3 | 0.33199295D 00 | 0.25406805D 00 | 0.11401790D-01 | 0.13867318D 00 | -0.79873628D-01 | 0.16569480D 00 |
| 4 | 0.48676135D 01 | 0.76931886D 01 | 0.38392642D 01 | 0.21631509D 01 | 0.11661093D 01 | 0.85586902D 00 |
| 5 | 0.19499045D 01 | 0.19685130D 01 | 0.18086748D 00 | 0.56103654D 00 | -0.31155904D 00 | 0.41249805D 00 |
| 6 | 0.22158191D 02 | 0.38255877D 02 | 0.13354481D 02 | 0.59455415D 01 | 0.23965909D 01 | 0.15473189D 01 |
| 7 | 0.12635598D 02 | 0.15453605D 02 | 0.18162968D 01 | 0.22628252D 01 | -0.12183169D 01 | 0.10759424D 01 |
| 8 | 0.11905467D 03 | 0.22532862D 03 | 0.53493170D 02 | 0.19278800D 02 | 0.61012404D 01 | 0.33637322D 01 |
| 9 | 0.88135037D 02 | 0.12597515D 03 | 0.15219404D 02 | 0.93665702D 01 | -0.50757369D 01 | 0.29991476D 01 |
| 10 | 0.71139685D 03 | 0.15068492D 04 | 0.23777097D 03 | 0.69822927D 02 | 0.18690194D 02 | 0.83500172D 01 |
| | **Moments About Midpoint** | | | | | |
| | Subtest 4 | Subtest 5 | Subtest 6 | Subtest 7 | Subtest 8 | Subtest 9 |
| 1 | -0.48482600D-01 | -0.45614000D-01 | -0.1936[illegible]400D 00 | -0.19436400D-01 | 0.27001640D 00 | -0.64152209D-01 |
| 2 | 0.14355313D 01 | 0.20019224D 01 | 0.14415415D 01 | 0.10374109D 01 | 0.83956563D 00 | 0.64228892D 00 |
| 3 | -0.10941253D 00 | -0.15640622D 00 | -0.81049030D 00 | 0.61205735D-02 | 0.58541585D 00 | -0.59677177D-01 |
| 4 | 0.48219461D 01 | 0.76798212D 01 | 0.41469497D 01 | 0.21508147D 01 | 0.14441850D 01 | 0.83093735D 00 |
| 5 | -0.52772519D 00 | -0.65707126D 00 | -0.36292759D 01 | 0.10200010D 00 | 0.14331390D 01 | -0.77819148D-01 |
| 6 | 0.21721646D 02 | 0.37986926D 02 | 0.15326503D 02 | 0.58608614D 01 | 0.32865236D 01 | 0.14302052D 01 |
| 7 | -0.30304234D 01 | -0.27559386D 01 | -0.17100406D 02 | 0.50540569D 00 | 0.38887129D 01 | -0.12492143D 00 |
| 8 | 0.11513414D 03 | 0.22186041D 03 | 0.64973744D 02 | 0.18807810D 02 | 0.87763387D 01 | 0.29242868D 01 |
| 9 | -0.19001617D 02 | -0.11158716D 02 | -0.83659341D 02 | 0.20844964D 01 | 0.11347348D 02 | -0.22655718D 00 |
| 10 | 0.67624060D 03 | 0.14677037D 04 | 0.30075101D 03 | 0.67390258D 02 | 0.25949567D 02 | 0.67664675D 01 |
| Number of Examinees | 500 | 500 | 500 | 500 | 500 | 498 |

TABLE 3-2

Coefficients, $\omega_j$ , of the Polynomials of Degrees 3 Through 7, Which Were Obtained by the Method of Moments, to Approximate the Density Function, $\hat{g}(\hat{\tau})$ , for Each of the Six Subtests.

| j | | Coefficient $\omega_j$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | Subtest 4 | Subtest 5 | Subtest 6 | Subtest 7 | Subtest 8 | Subtest 9 |
| 0 | DGR. 3 | 0.280688[illegible] | 0.23710680D+00 | 0.27675041D+00 | 0.32989144D+00 | 0.37924040D+00 | 0.42403040D+00 |
| 1 | | -0.4802[illegible]885D-02 | -0.40260384D-02 | 0.72069596D-02 | -0.11139437D-01 | 0.23362577D-01 | -0.31365098D-01 |
| 2 | | -0.39530649D-01 | -0.23685535D-01 | -0.34926777D-01 | -0.64056925D-01 | -0.94378124D-01 | -0.13877014D+00 |
| 3 | | 0.20722819D-02 | 0.86421644D-03 | -0.32393809D-02 | 0.45084767D-02 | -0.94127572D-02 | 0.24177672D-0[illegible] |
| 0 | DGR. 4 | 0.30042586D+00 | 0.21343248D+00 | 0.26078376D+00 | 0.31154572D+00 | 0.36757665D+00 | 0.40299049D+00 |
| 1 | | 0.67956150D-03 | -0.74730099D-02 | -0.32140928D-02 | -0.14[illegible]37947D-01 | 0.43086179D-01 | -0.47557111D-01 |
| 2 | | -0.66071432D-01 | 0.14716600D-02 | -0.87525684D-02 | -0.28[illegible]78862D-01 | -0.62689731D-01 | -0.71262116D-01 |
| 3 | | 0.35374619D-03 | 0.17186031D-02 | 0.72906955D-03 | 0.58[illegible]97537D-02 | -0.21683836D-01 | 0.36261431D-01 |
| 4 | | 0.41874652D-02 | -0.31250428D-02 | -0.51298480D-02 | -0.788[illegible]5931D-02 | -0.10928998D-01 | -0.25721070D-01 |
| 0 | DGR. 5 | 0.30234305D+00 | 0.21339774D+00 | 0.25601295D+00 | 0.31217849D+00 | 0.34514841D+00 | 0.40645552D+00 |
| 1 | | -0.17759677D-01 | -0.69671678D-02 | 0.20043890D-01 | -0.28961057D-01 | -0.20963377D-01 | -0.75841476D-01 |
| 2 | | -0.69673897D-01 | 0.15233010D-02 | 0.21042253D-02 | -0.30576762D-01 | 0.19905676D-01 | -0.86733084D-01 |
| 3 | | 0.11958256D-01 | 0.14675073D-02 | -0.17294924D-01 | 0.19123261D-01 | 0.63859509D-01 | 0.79003567D-01 |
| 4 | | [illegible].49140991D-02 | -0.31332741D-02 | -0.77985278D-02 | -0.73922332D-02 | -0.44432685D-01 | -0.18276505D-01 |
| 5 | | -0.14164404D-02 | 0.24065806D-04 | 0.27597516D-02 | -0.22954640D-02 | -0.23871526D-01 | -0.12676995D-01 |
| 0 | DGR. 6 | 0.30252153D+00 | 0.19087739D+00 | 0.24336441D+00 | 0.28339981D+00 | 0.32645677D+00 | 0.36748907D+00 |
| 1 | | -0.17654561D-01 | -0.13878290D-01 | 0.18454616D-02 | -0.38864235D-01 | 0.60829257D-01 | -0.14100846D+00 |
| 2 | | -0.70177475D-01 | 0.51778851D-01 | 0.45674854D-01 | 0.85309368D-01 | 0.12766974D+00 | 0.17590607D+00 |
| 3 | | 0.11873806D-01 | 0.58725679D-02 | 0.52789384D-03 | 0.[illegible]0514568D-01 | -[illegible].67114847D-01 | 0.20407473D+00 |
| 4 | | 0.51177376D-02 | -0.19186744D-01 | -0.29797514D-01 | -0.74150743D-01 | -0.14259277D+00 | -0.27583372D+00 |
| 5 | | -0.14039381D-02 | -0.49099147D-03 | -0.46615442D-03 | -0.46987407D-02 | 0.15650408D-01 | -0.56942346D-01 |
| 6 | | -0.20309040D-04 | 0.12559797D-02 | 0.27799948D-02 | 0.94135397D-02 | 0.23466295D-01 | 0.62814462D-01 |
| 0 | DGR. 7 | 0.30515755D+00 | 0.19107767D+00 | 0.24641184D+00 | 0.28332638D+00 | 0.30655929D+00 | 0.36746142D+00 |
| 1 | | -0.42690648D-01 | -0.16782234D-01 | -0.12155795D-01 | -0.37149298D-01 | 0.18151090D-01 | -0.14079194D+00 |
| 2 | | -0.79730014D-01 | 0.51204621D-01 | 0.32300476D-01 | 0.85689345D-01 | 0.26906961D+00 | 0.17614429D+00 |
| 3 | | 0.42262785D-01 | 0.86525981D-02 | 0.21484259D-01 | 0.27553026D-01 | 0.46243769D-01 | 0.20344279D+00 |
| 4 | | 0.98277279D-02 | -0.18962999D-01 | -0.21759416D-01 | -0.74417913D-01 | -0.28301912D+00 | -0.27611346D+00 |
| 5 | | -0.10472358D-01 | -0.11428754D-02 | -0.83333809D-02 | -0.34467281D-02 | -0.65386797D-01 | -0.56484398D-01 |
| 6 | | -0.57024974D-03 | 0.12353608D-02 | 0.16290614D-02 | 0.94579597D-02 | 0.57340888D-01 | 0.62892827D-01 |
| 7 | | 0.76572083D-03 | 0.43095269D-04 | 0.85015024D-03 | -0.14913539D-03 | 0.17239856D-01 | -0.95316339D-04 |

of coefficients, $\omega_j$ , of the polynomials of degrees 3 through 7 to approximate the marginal density function, $g(\hat{\tau})$ , which are given by

$$(3.1) \qquad \hat{g}(\hat{\tau}) = \sum_{j=0}^{p} \omega_j \hat{\tau}^j , \qquad p=3,4,5,6,7 ,$$

for each of the six Old Tests.

Figure 3-1 presents these resultant five polynomials approximating the density function, $g(\hat{\tau})$ , for each of our six Old tests. In the same figure, also presented is the set of $\hat{\tau}_s$ 's , in the form of a frequency distribution with the subinterval width of 0.25 . We can see in this figure that Subtest 5 has the smoothest frequency distribution, and, as we proceed up to Subtest 9, the frequency distribution becomes less smooth. This phenomenon is the result of the reduction in the amount of test information, as we can see in Figure 2-1 in the preceding chapter. We can also see in Figure 3-1 that the frequency distribution of Subtest 4 has a high irregularity around $\hat{\tau} = 0.0$ , and this is again due to the small amount of test information around $\theta = 0.0$ , as is shown in Figure 2-1.

We have observed with Subtest 2 (Samejima, RR-80-4) that, regardless of the high irregularity of the frequency distribution of $\hat{\tau}_s$ and the resultant low degree of fitness of the polynomial, the estimation of the item characteristic functions turned out to be quite successful. For this reason, without trying to increase the degree of the polynomial nor using several polynomials for the subintervals to increase the fitness, we simply adopted the polynomial of degree 4 in each of the six situations. The choice of the degree 4 over the others is based upon the fact that in the previous studies we compared the results obtained by using the polynomials of degrees 3 and 4 , and sometimes included the one obtained by using the polynomial of degree 5, and the resultant estimated item characteristic functions in Degree 4 case turned out to be just as good as we could wish for. These polynomials of degree 4 are shown as larger graphs in Figure 3-1 for the six Old Tests.

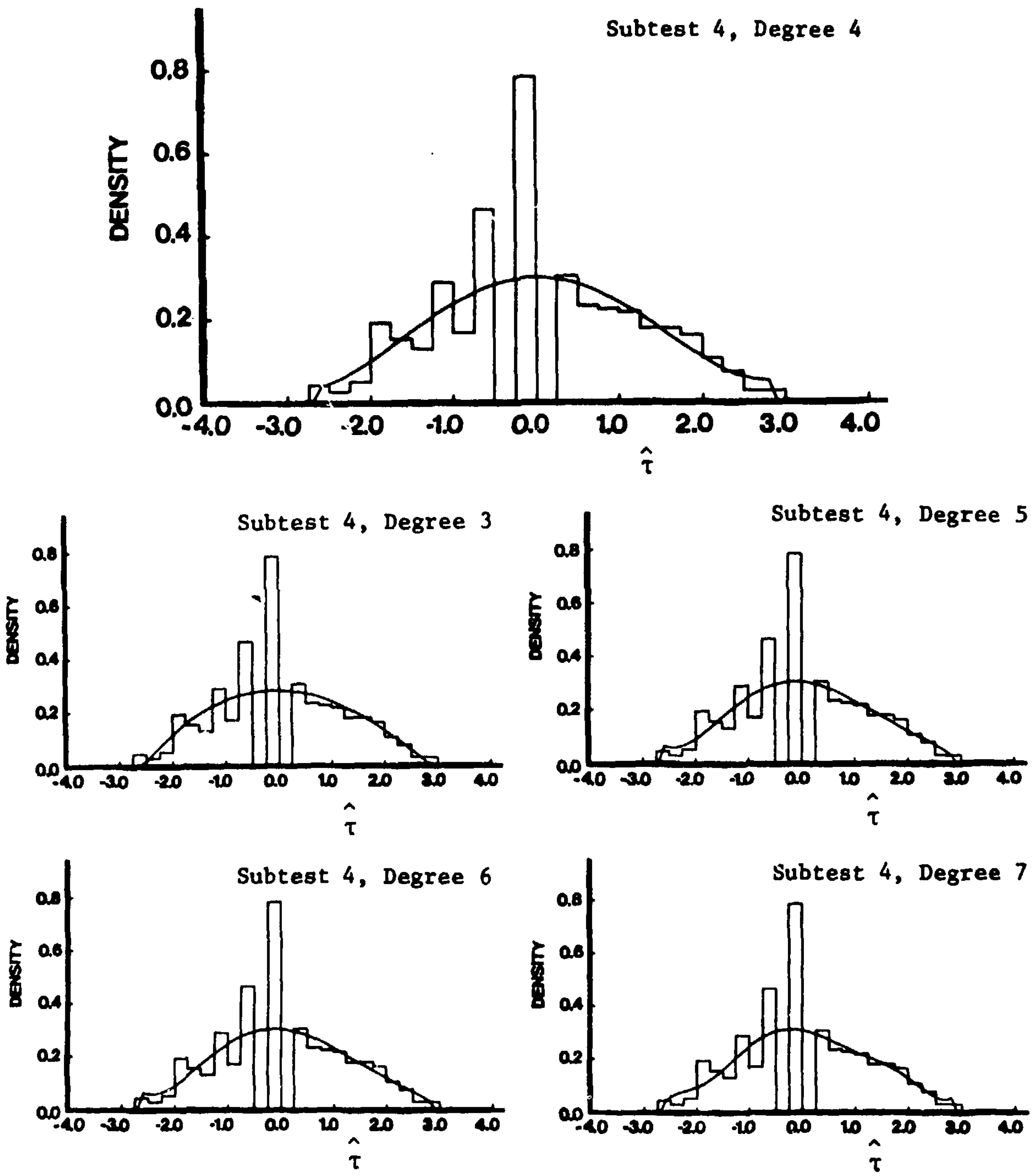



FIGURE 3-1

Estimated Density Function, $\hat{g}(\hat{\tau})$ , Obtained by the Method of Moments as a Polynomial, Together with the Relative Frequency Distribution of the Five Hundred $\hat{\tau}$'s , for Each of the Six Subtests. The Degrees of the Five Polynomials Are 3, 4, 5, 6 and 7.

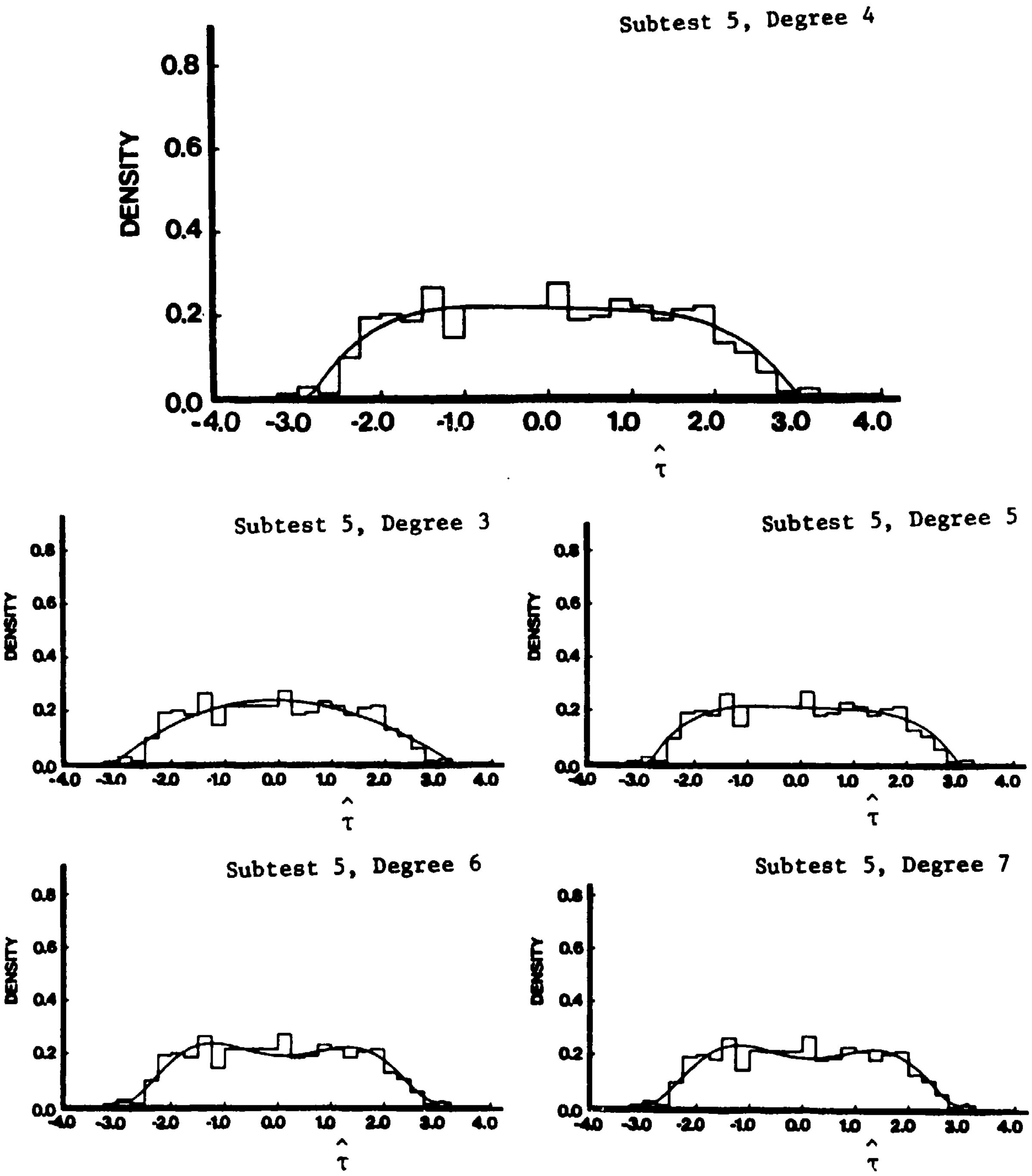



FIGURE 3-1 (Continued)

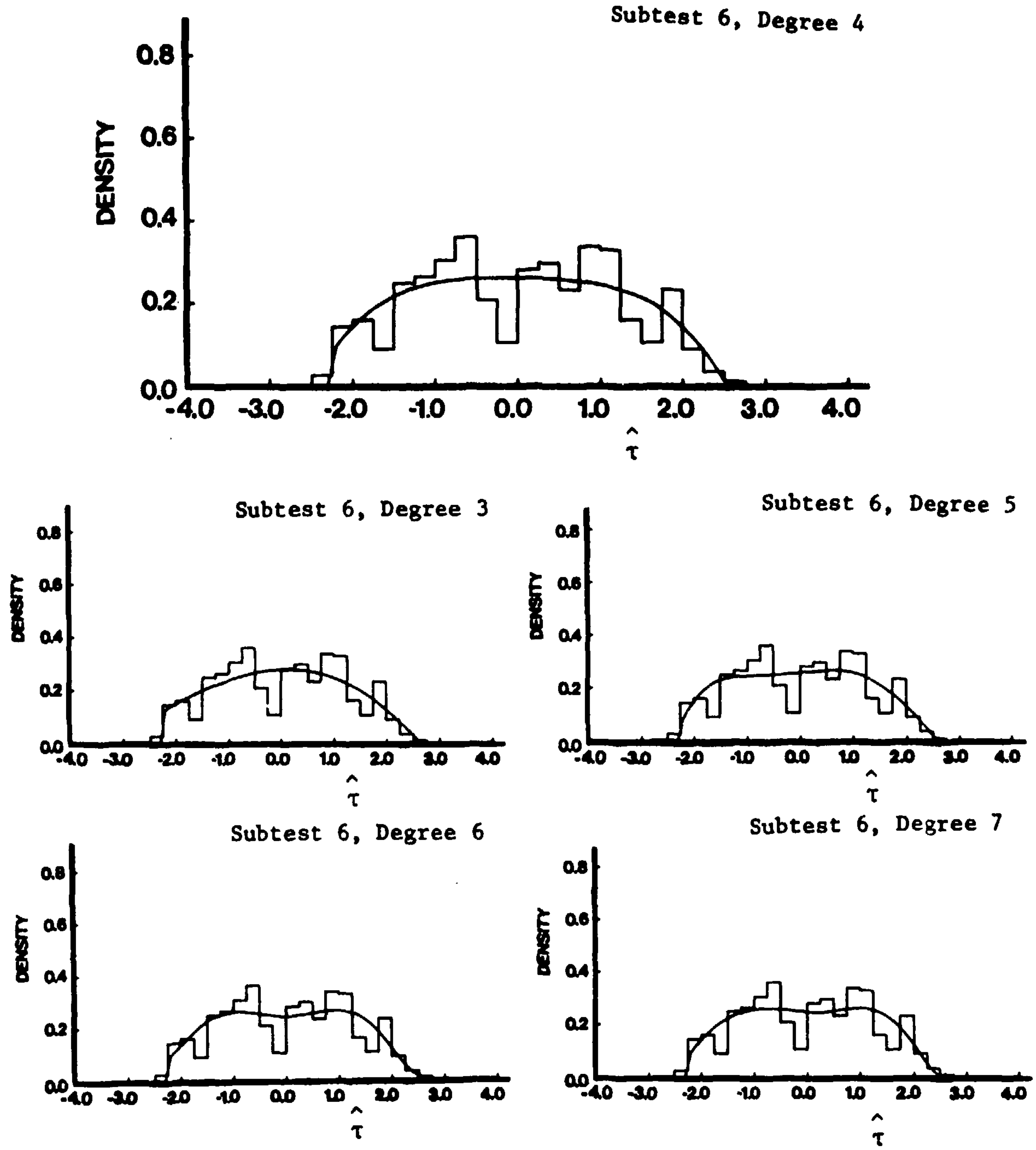



FIGURE 3-1 (Continued)

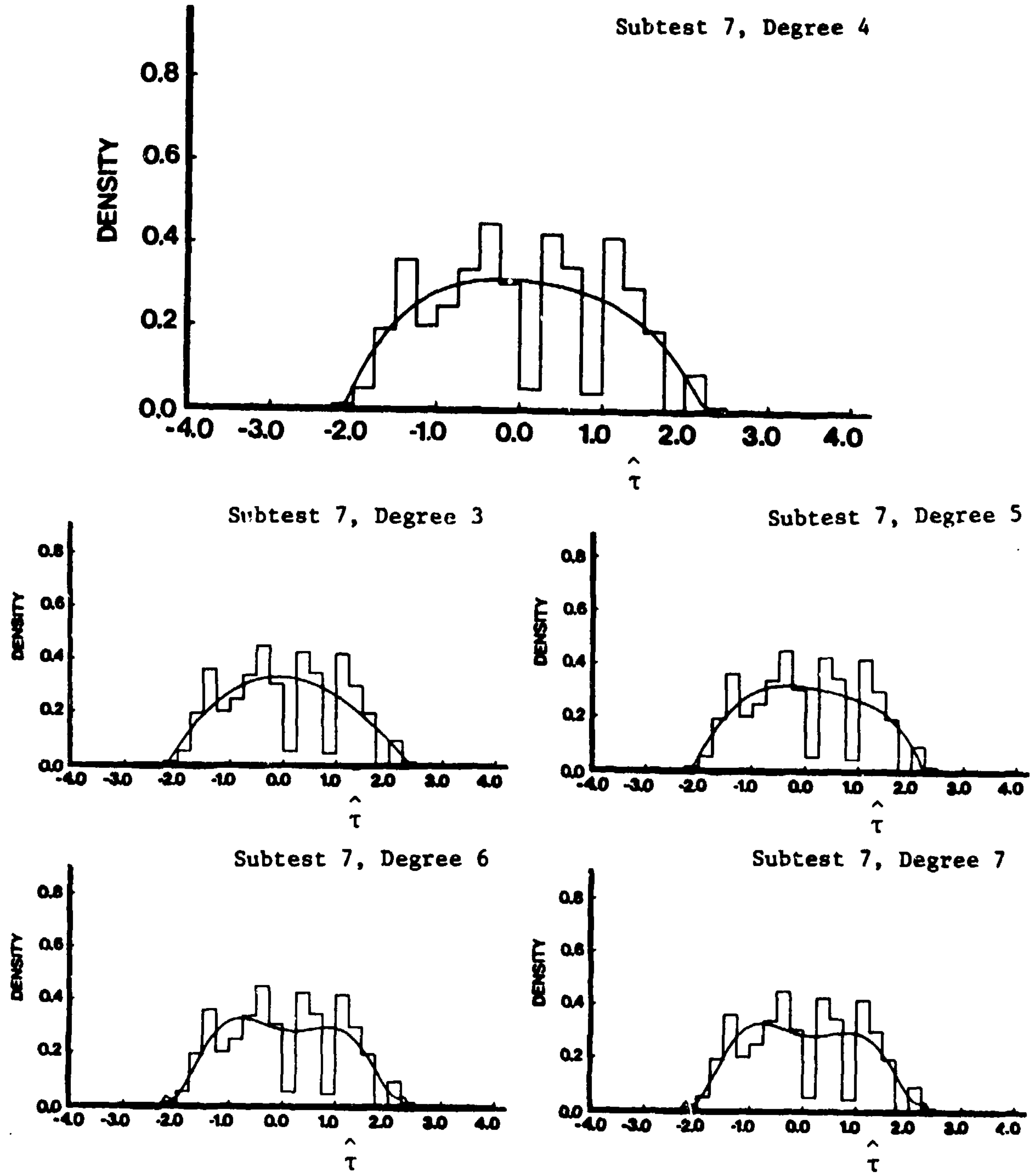



FIGURE 3-1 (Continued)

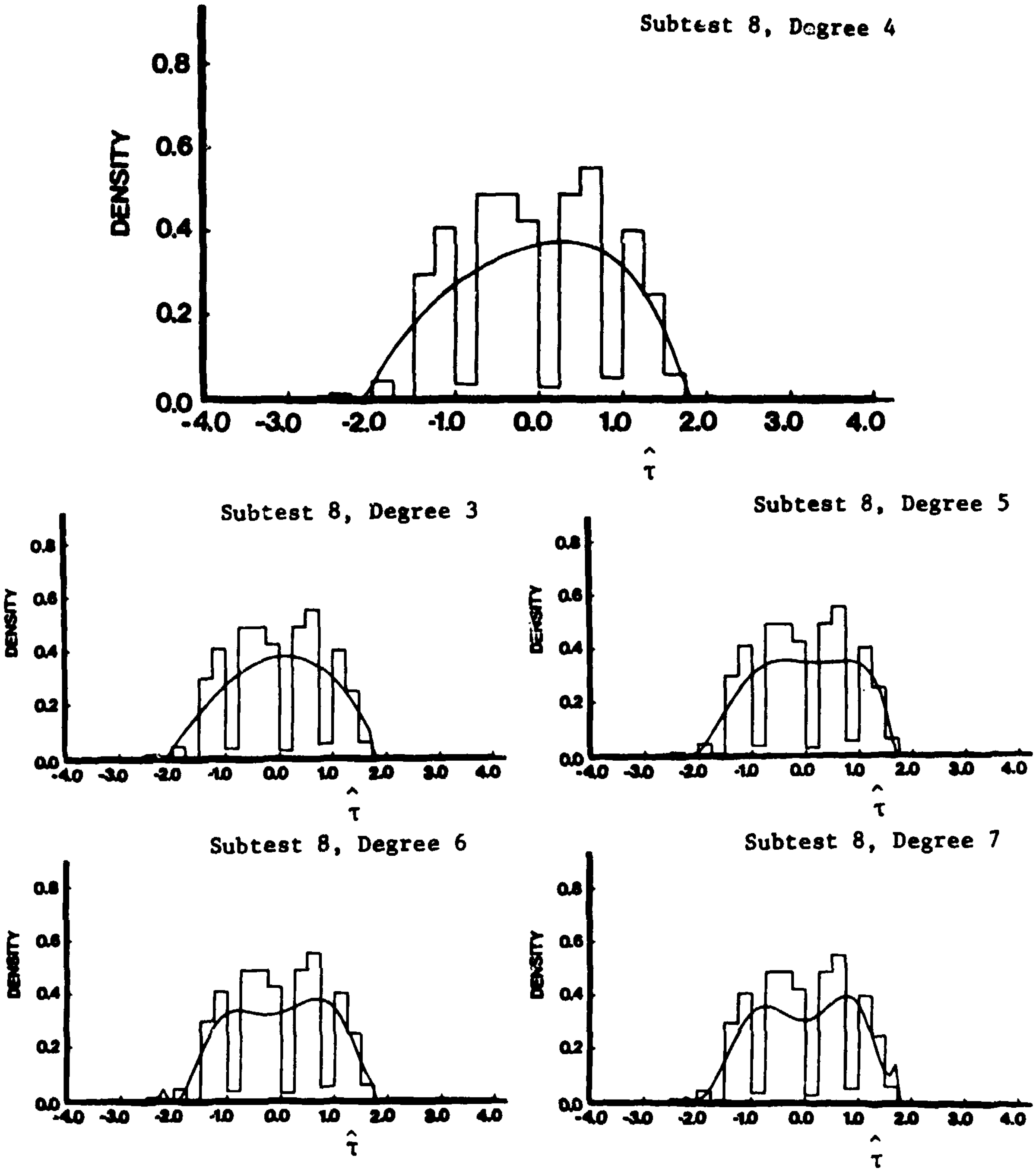
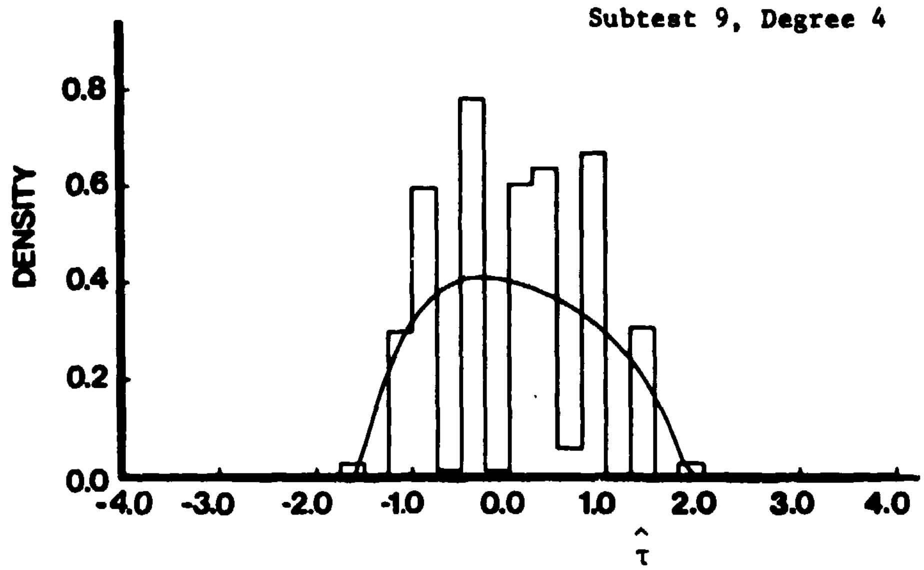



FIGURE 3-1 (Continued)



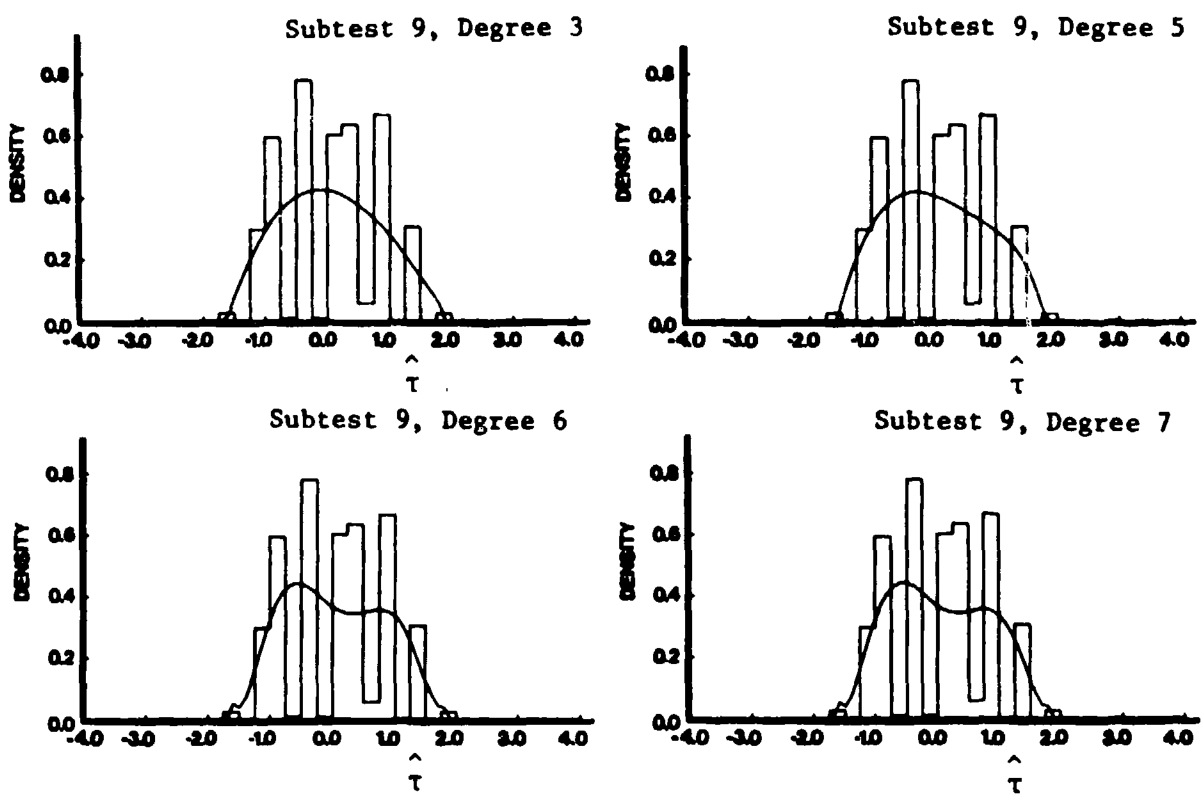




FIGURE 3-1 (Continued)

## IV Estimation of the Item Characteristic Functions of the Ten Binary Test Items

As was the case with the previous studies (Samejima, RR-77-1, RR-78-1, RR-78-2, RR-78-3, RR-78-4, RR-78-5, RR-78-6, RR-80-4, RR-81-2), ten binary test items, which follow the normal ogive model given by (2.2) and whose parameters, $a_h$ and $b_h$, are shown in Table 4-1, were chosen as unknown test items, whose item characteristic functions, i.e., $P_{x_h}(\theta)$ for $x_h = m_h = 1$, $h=1,2,...,10$, are to be estimated.

We obtained the estimates of the conditional expectation and variance of $\tau$, given $\hat{t}$, which are given by

$$(4.1) \qquad \hat{E}(\tau|\hat{t}) = \hat{t} + C^{-2}\frac{d}{d\hat{t}} \log \hat{g}(\hat{t})$$

and

$$(4.2) \qquad \hat{Var}.(\tau|\hat{t}) = C^{-2}[1 + C^{-2}\frac{d^2}{d\hat{t}^2} \log \hat{g}(\hat{\tau})]$$

where $C$ is the target square root of the test information, $[I^*(\tau)]^{1/2}$, and $\hat{g}(\hat{t})$ is the polynomial of degree 4 obtained by the method of moments as the estimate of the density function, $g(\hat{t})$, for each of the six sets of $\hat{t}_s$'s obtained upon Subtests 4 through 9. In so doing we found that for some examinees the estimated conditional density, $\hat{g}(\hat{t}_s)$, assumed negative values, and they have to be excluded from the original set of data. They number five for Subtest 5, one for Subtest 6, two for Subtest 7, one for Subtest 8 and six for Subtest 9. We also found that for some other examinees the estimated conditional variance, which is given by (4.2), turned out to be negative values. There are two such examinees for Subtest 5, eight for Subtest 6, seventeen for Subtest 7, and twelve for Subtest 8. As the result, the total numbers of hypothetical examinees who are included in our data are: 500, 493, 491, 481, 487 and 492, for Subtests 4 through 9, respectively.

From the first two conditional moments of $\tau$ about the origin we approximated the conditional density function, $\phi(\tau|\hat{t})$, of $\tau$, given

TABLE 4-1

Item Discrimination Parameter, $a_h$,
and Item Difficulty Parameter, $b_h$,
of Each of Ten Binary Items.

| Item h | $a_h$ | $b_h$ |
|---|---|---|
| 1 | 1.5 | -2.5 |
| 2 | 1.0 | -2.0 |
| 3 | 2.5 | -1.5 |
| 4 | 1.0 | -1.0 |
| 5 | 1.5 | -0.5 |
| 6 | 1.0 | 0.0 |
| 7 | 2.0 | 0.5 |
| 8 | 1.0 | 1.0 |
| 9 | 2.0 | 1.5 |
| 10 | 1.0 | 2.0 |

$\hat{t}$, following the Normal Approach Method of the Conditional P.D.F. Approach. Thus we have

(4.3) $$\hat{\phi}(\tau|\hat{t}) = (2\pi\hat{\mu}_2)^{-1/2} \exp\{-(\tau-\hat{\mu}_1')^2/(2\hat{\mu}_2)\} \quad ,$$

where

(4.4) $$\hat{\mu}_1' = \hat{E}(\tau|\hat{t})$$

and

(4.5) $$\hat{\mu}_2 = \hat{V}ar.(\tau|\hat{t})$$

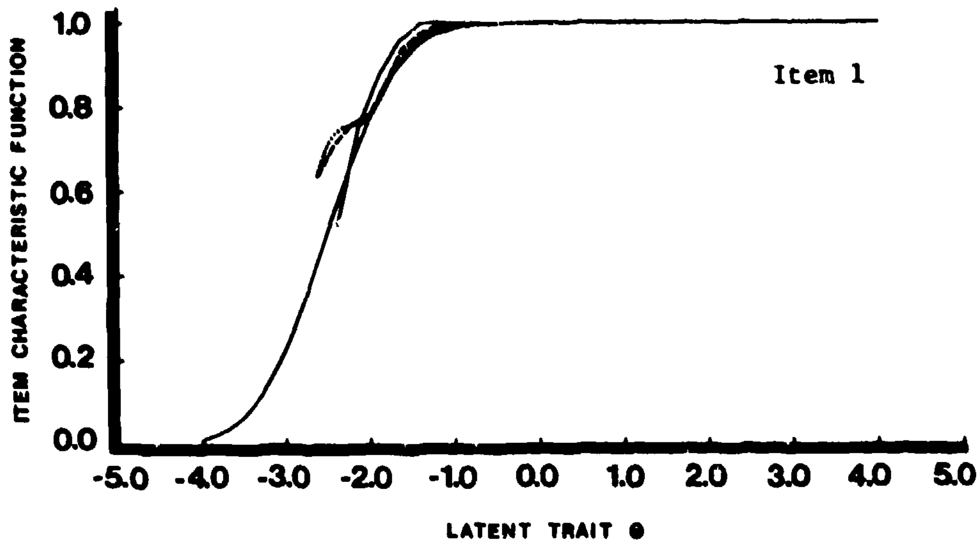
We adopted Simple Sum Procedure of the Conditional P.D.F. Approach to estimate the item characteristic functions of the ten binary test items, as we did for Subtests 1, 2 and 3 (cf. RR-80-4, RR-81-2). Let $\hat{P}_{x_h}(\theta)$ be the estimated operating characteristic of the item score $x_h$ $(=0,1,\ldots,m_h)$ of item $h$. We can write

(4.6) $$\hat{P}_{x_h}(\theta) = \hat{P}_{x_h}[\tau(\theta)] = \sum_{s\varepsilon x_h} \hat{\phi}(\tau|\hat{t}_s)[\sum_{s=1}^{n} \hat{\phi}(\tau|\hat{t}_s)]^{-1} \quad ,$$

where $\hat{\phi}(\tau|\hat{t}_s)$ is the estimated conditional density obtained by the Normal Approach Method, which is given by (4.3).

Figures 4-1 through 4-6 present the resultant estimated item characteristic functions for the ten binary test items by dotted curves, which are based upon Subtests 4 through 9, respectively. In these figures, also presented are the corresponding estimated item characteristic functions which were obtained upon the original Old Test by dashed curves, the theoretical item characteristic functions by solid curves and the frequency ratios of the correct answer obtained by using subintervals of $\theta$ with the width of 0.25 by jagged solid curves.

Comparison of these six sets of the estimated item characteristic functions indicates that, as the number of test items in the Old Test decreases, the accuracy of estimation also decreases. We can see in



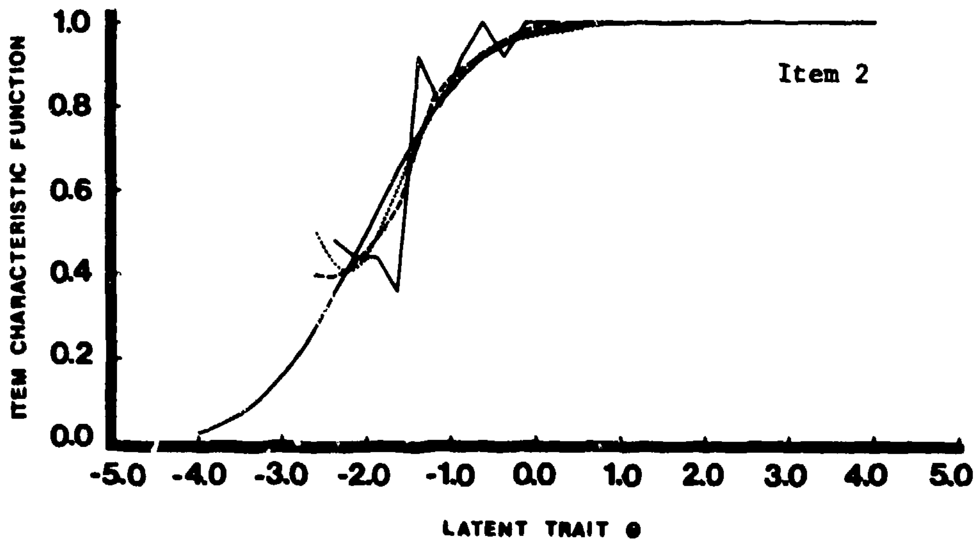
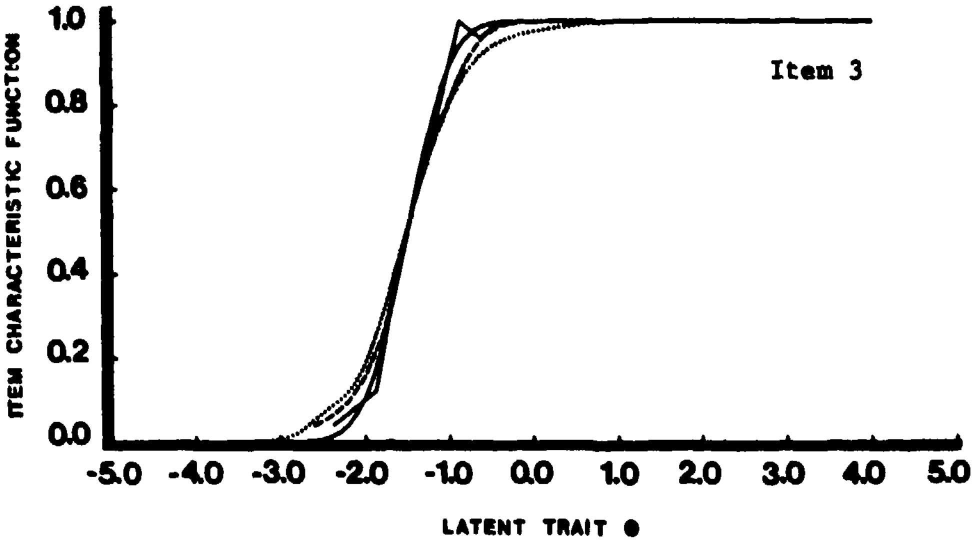




FIGURE 4-1

Estimated Item Characteristic Function Based upon Subtest 4 (Dotted Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method, for Degree 4 Case, in Comparison with the One Based upon the Original Old Test (Dashed Curve), the Theoretical Item Characteristic Function (Smooth Solid Curve) and the Frequency Ratios of Those Who Answered Correctly (Jagged Solid Line).



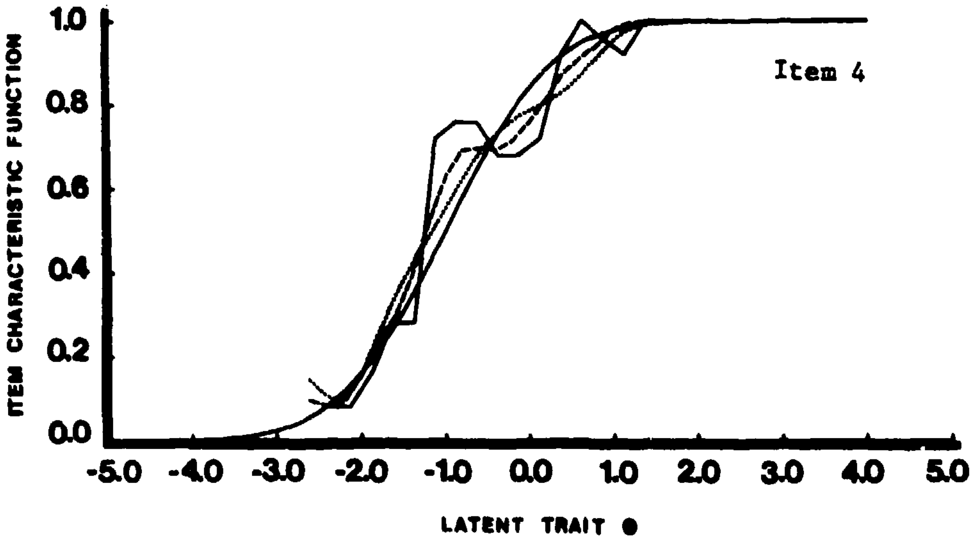




FIGURE 4-1 (Continued)

Subtest 4

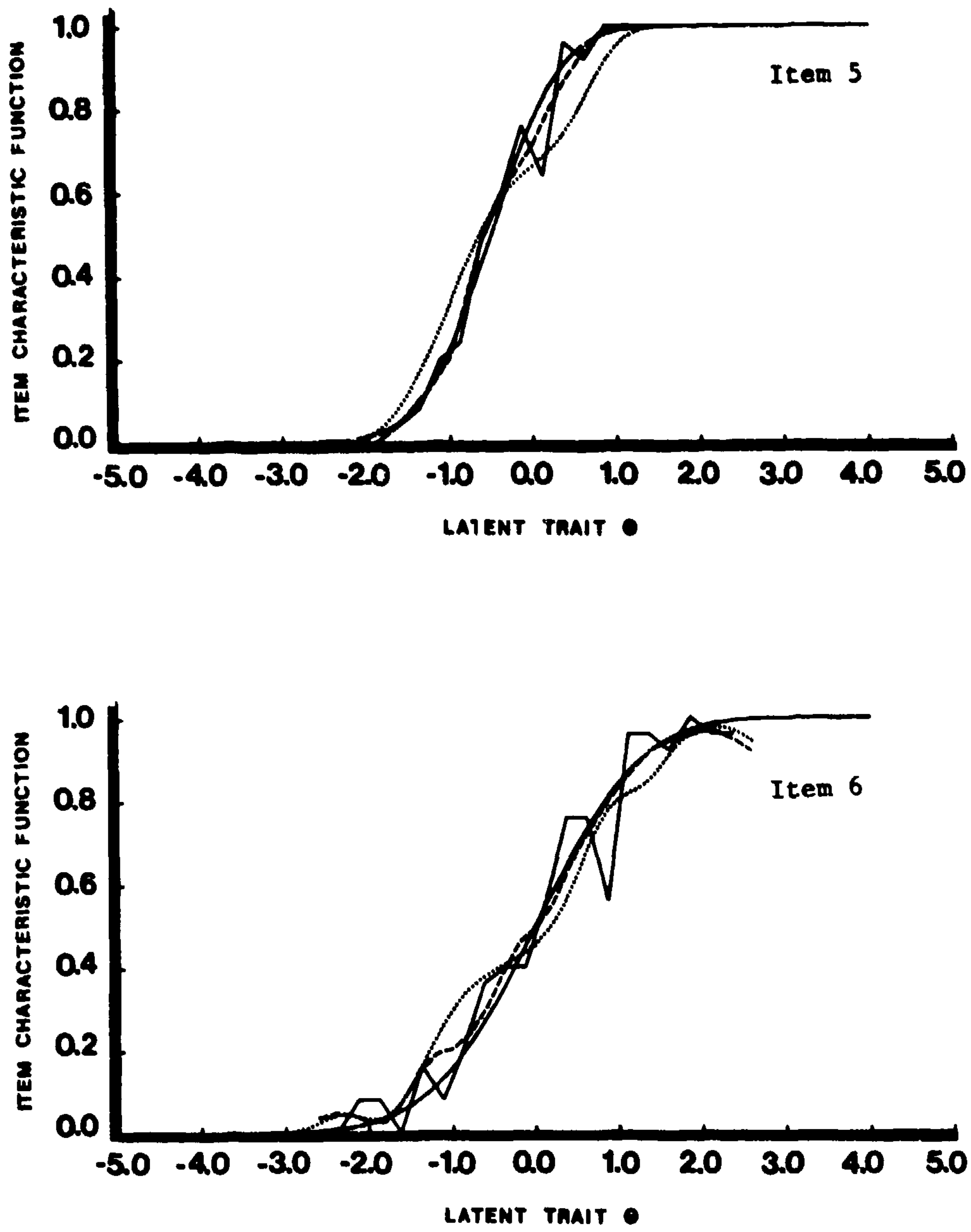
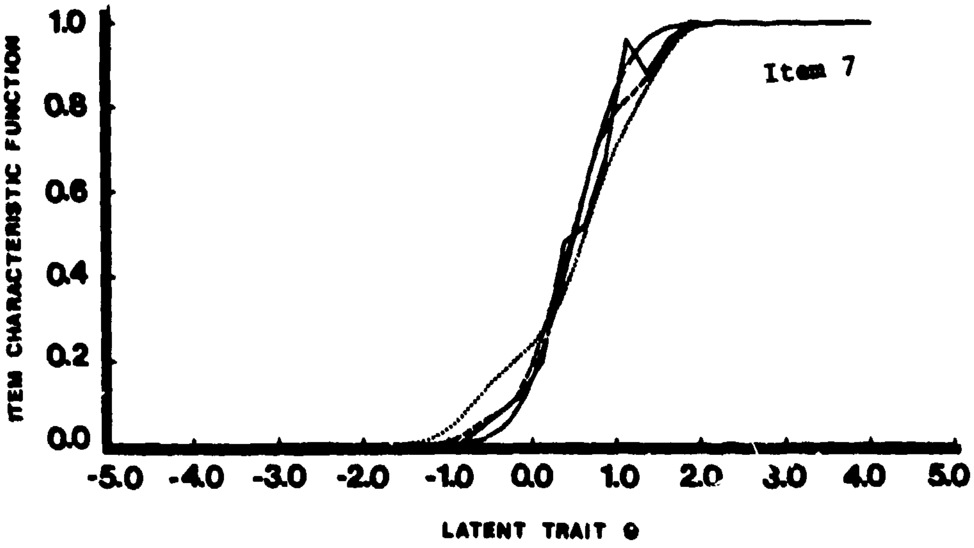



FIGURE 4-1 (Continued)

Subtest 4



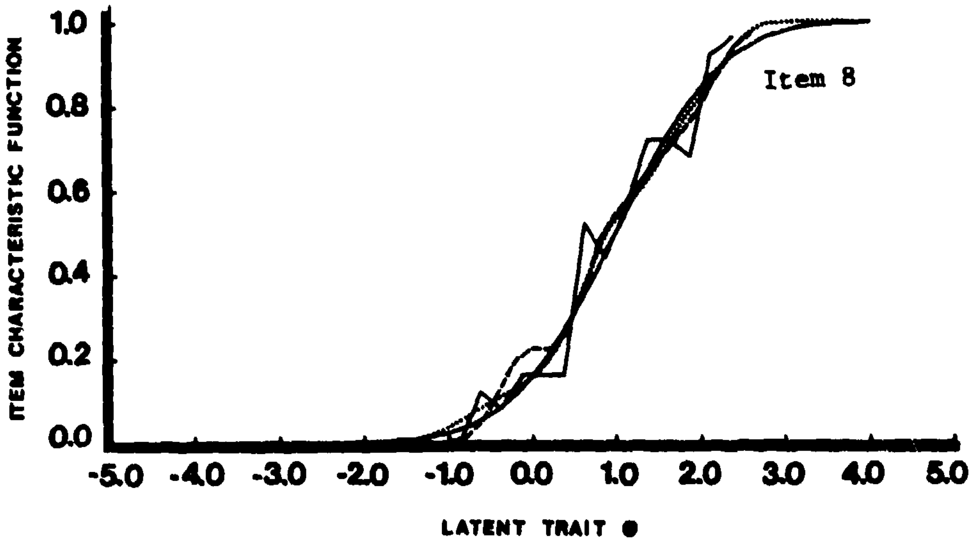
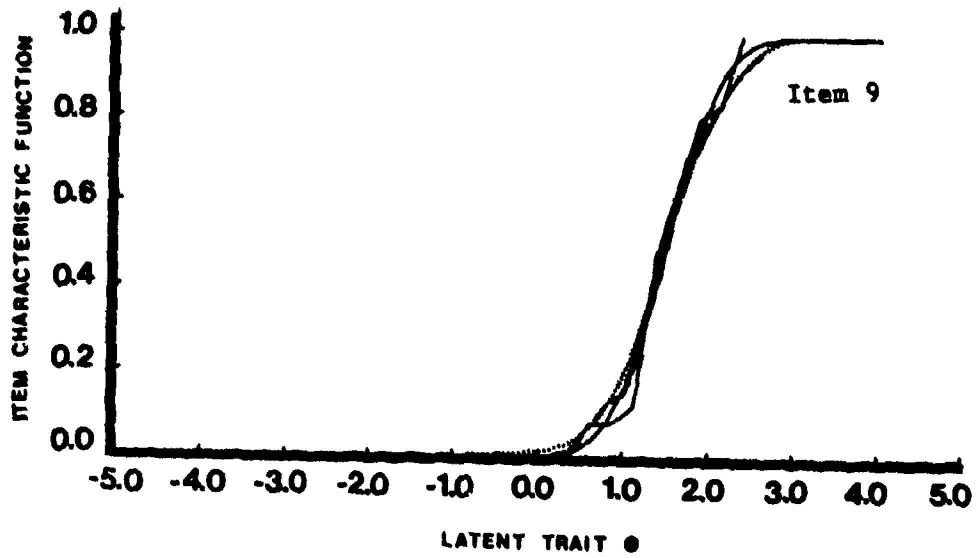




FIGURE 4-1 (Continued)

Subtest 4



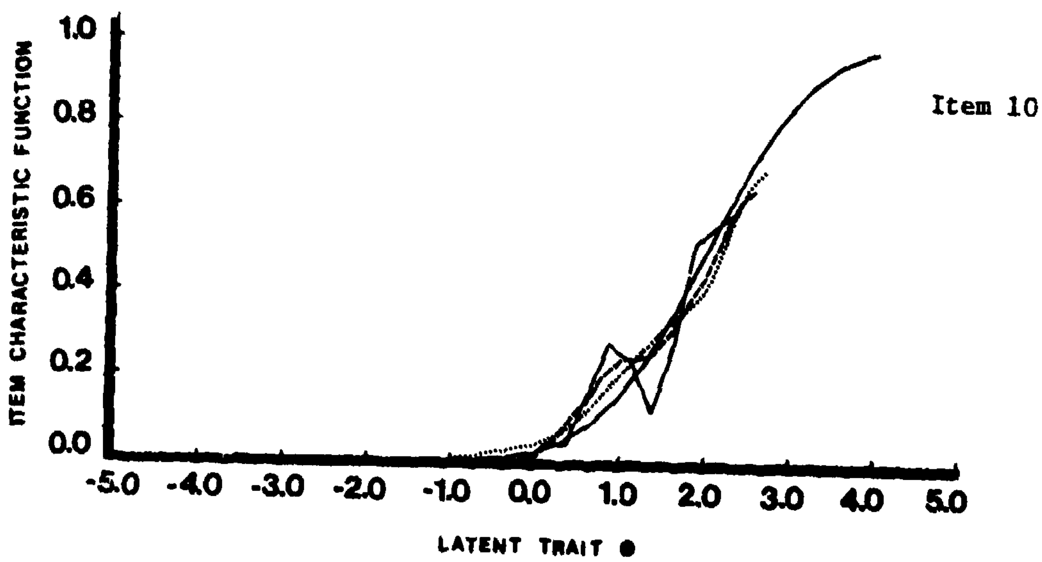




FIGURE 4-1 (Continued)
Subtest 4

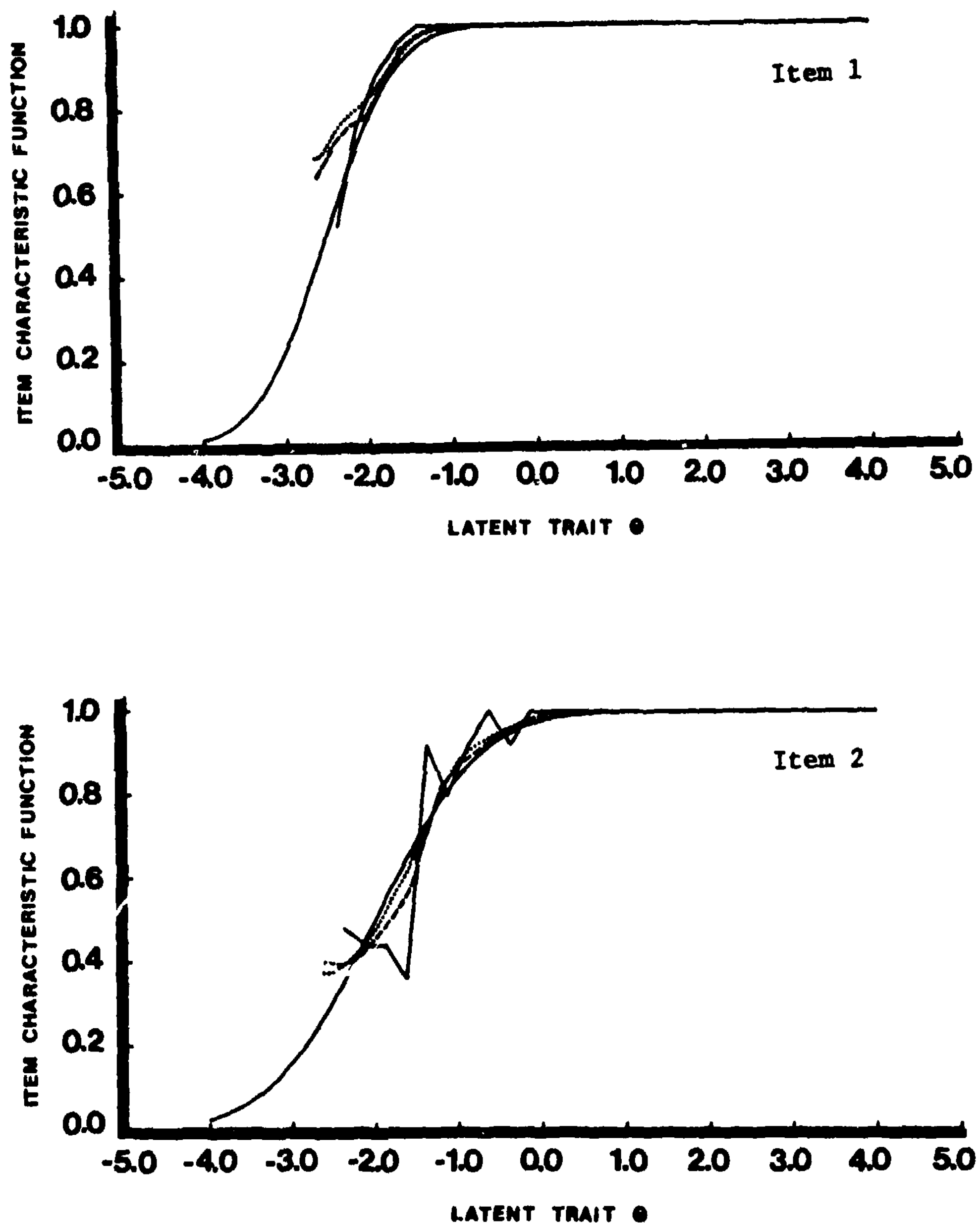
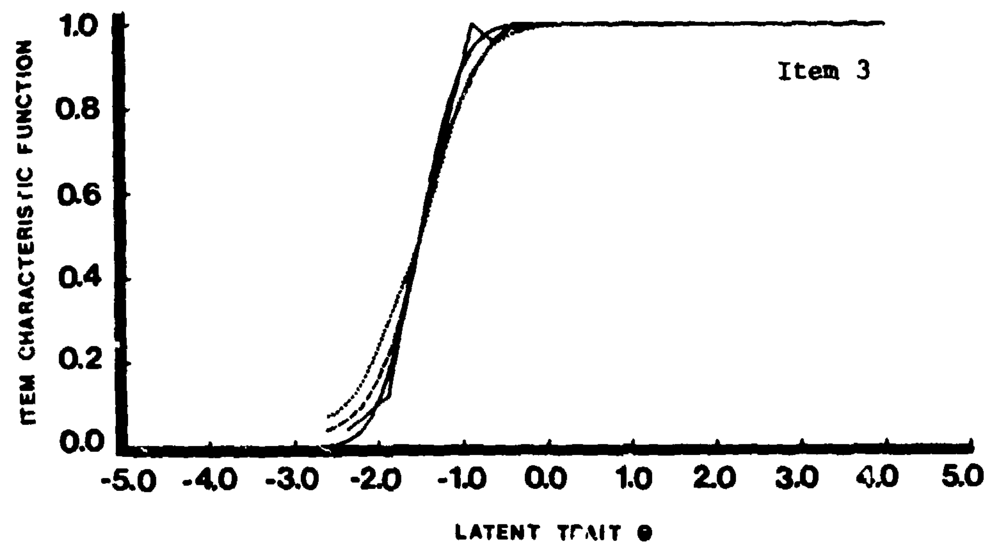



FIGURE 4-2

Estimated Item Characteristic Function Based upon Subtest 5 (Dotted Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method, for Degree 4 Case, in Comparison with the One Based upon the Original Old Test (Dashed Curve), the Theoretical Item Characteristic Function (Smooth Solid Curve) and the Frequency Ratios of Those Who Answered Correctly (Jagged Solid Line).



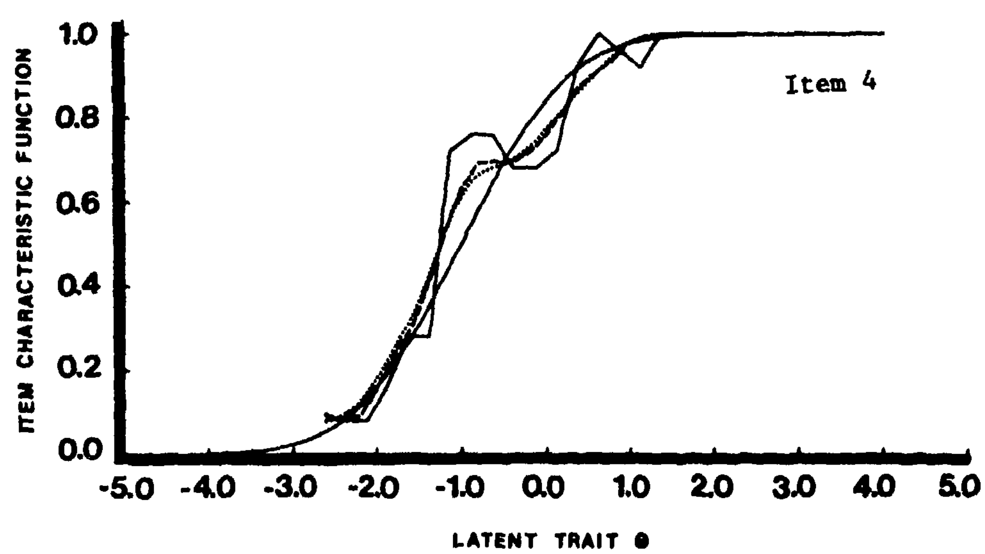
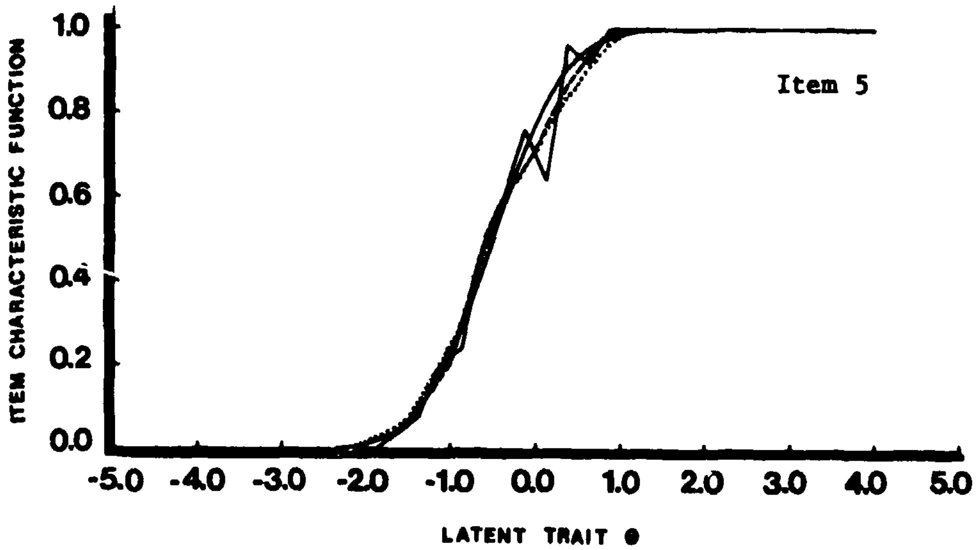




FIGURE 4-2 (Continued)

Subtest 5



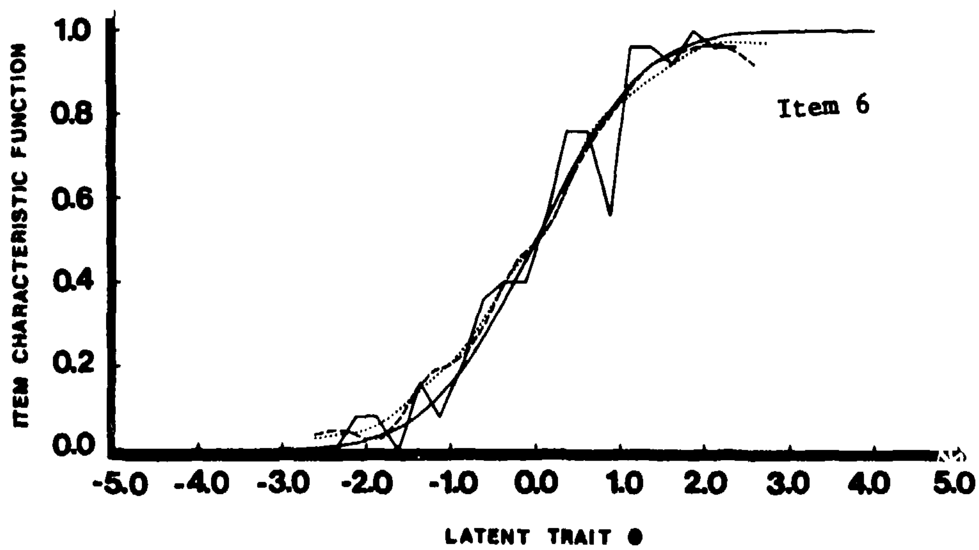
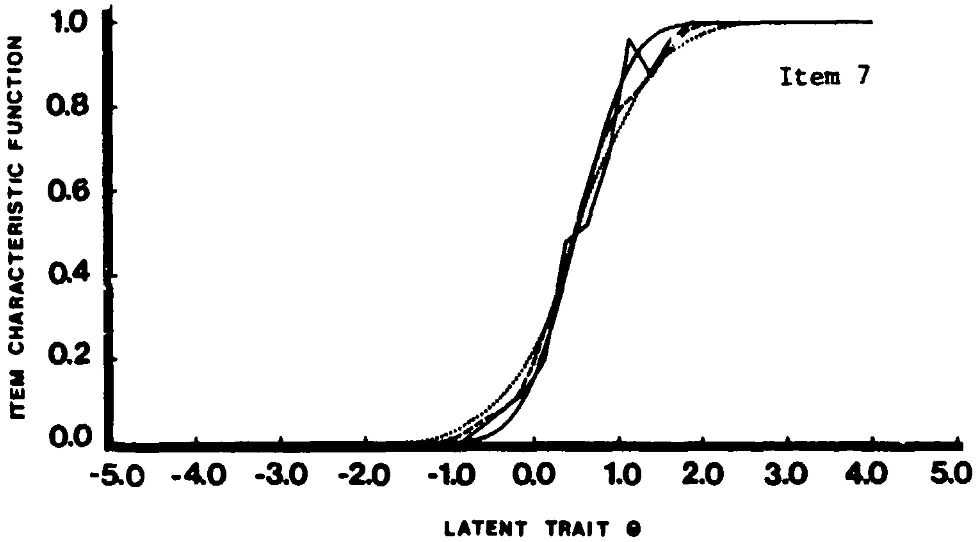




FIGURE 4-2 (Continued)
Subtest 5



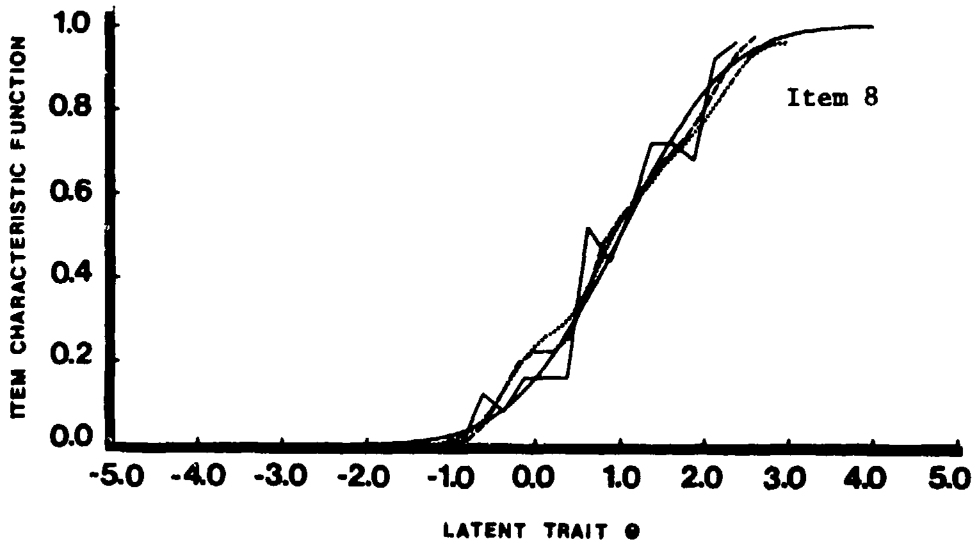
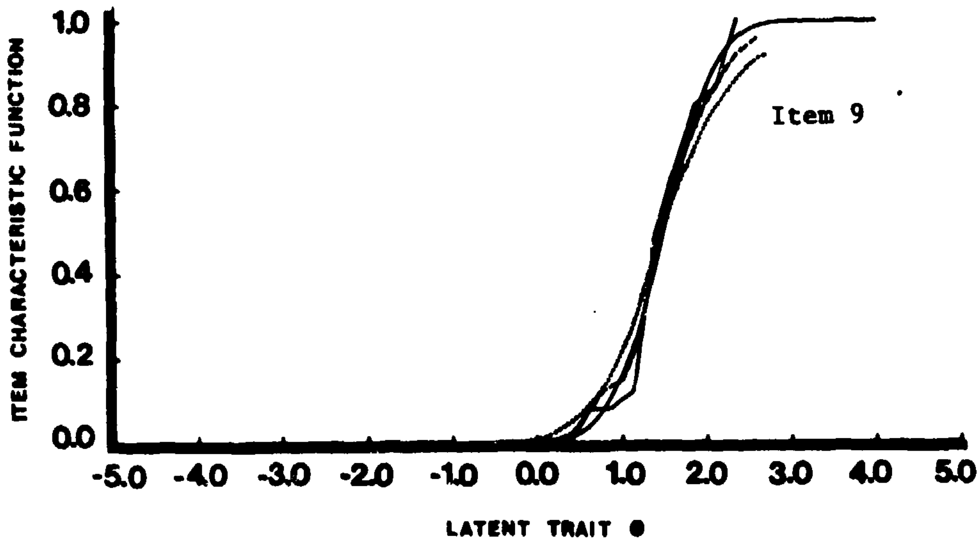




FIGURE 4-2 (Continued)
Subtest 5



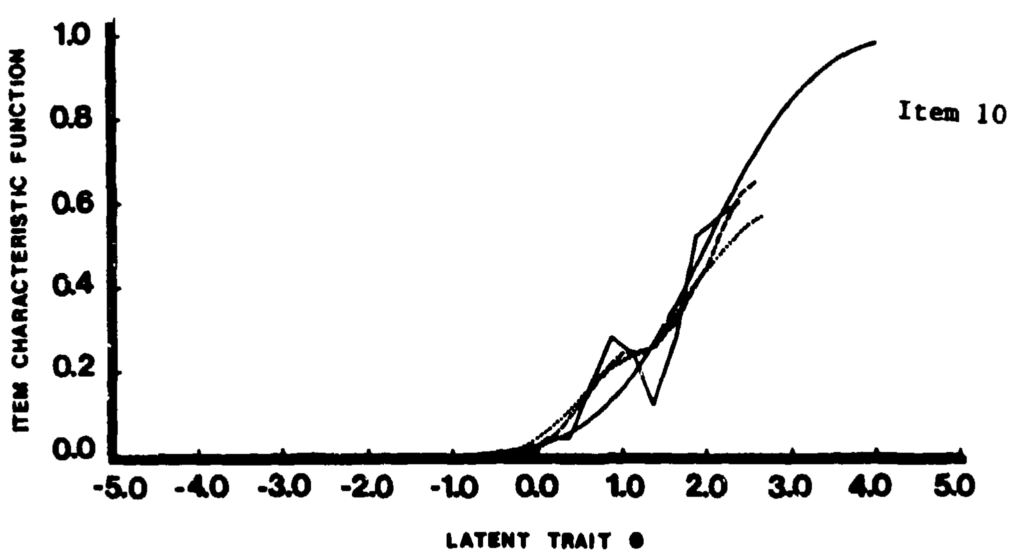
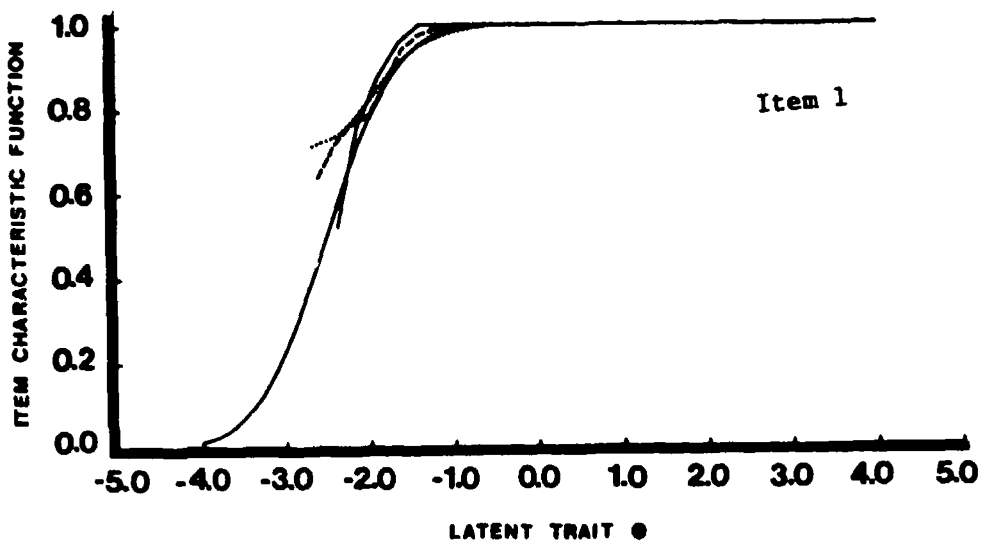




FIGURE 4-2 (Continued)
Subtest 5



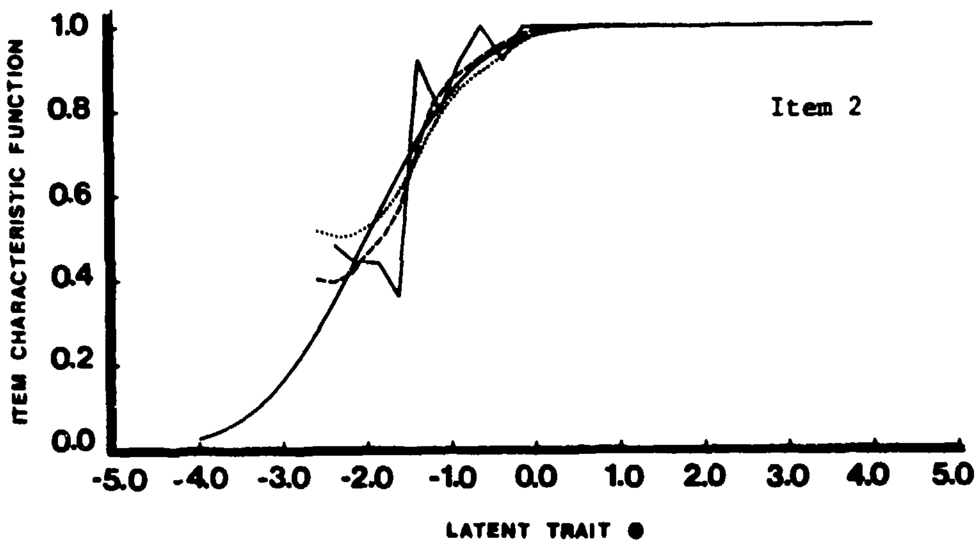
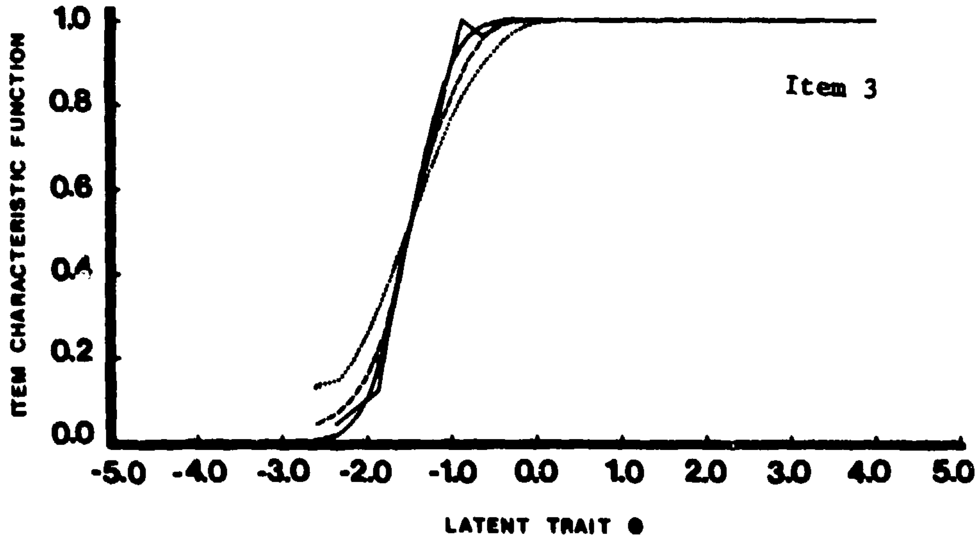




FIGURE 4-3

Estimated Item Characteristic Function Based upon Subtest 6 (Dotted Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method, for Degree 4 Case, in Comparison with the One Based upon the Original Old Test (Dashed Curve), the Theoretical Item Characteristic Function (Smooth Solid Curve) and the Frequency Ratios of Those Who Answered Correctly (Jagged Solid Line).



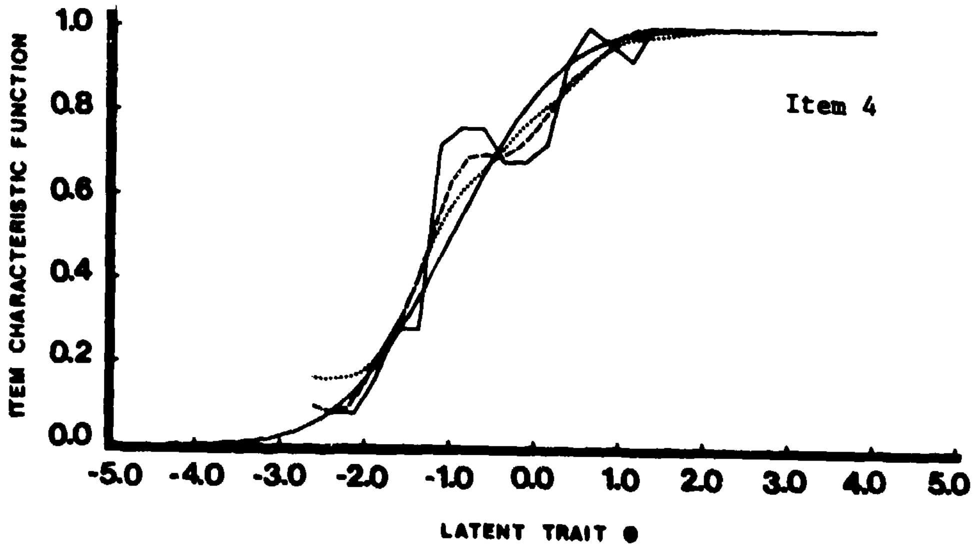
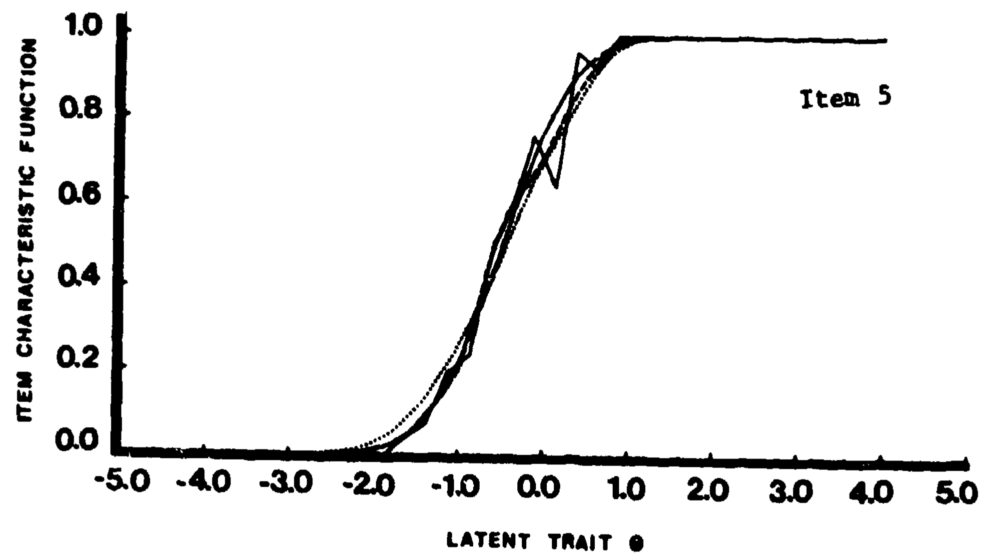




FIGURE 4-3 (Continued)
Subtest 6



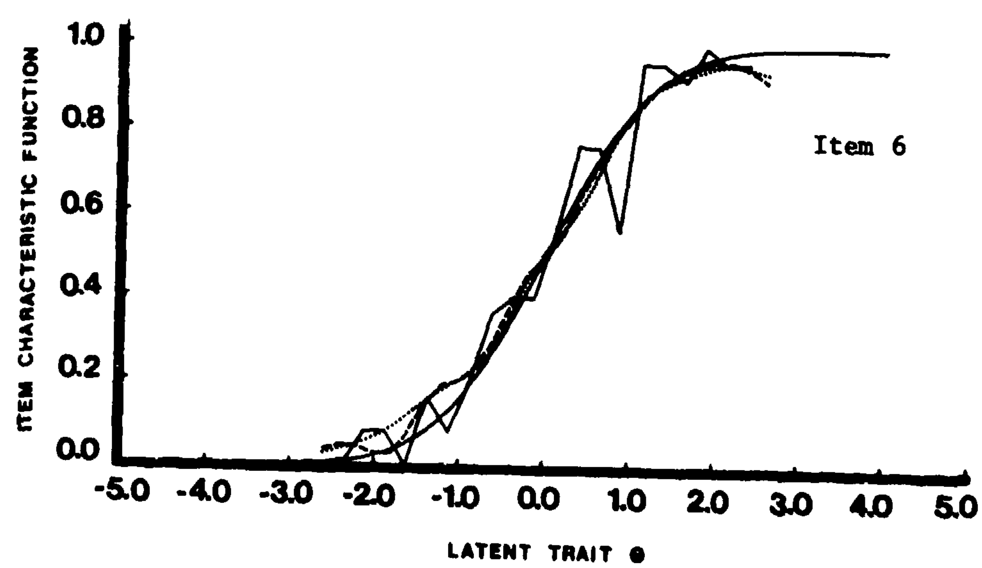
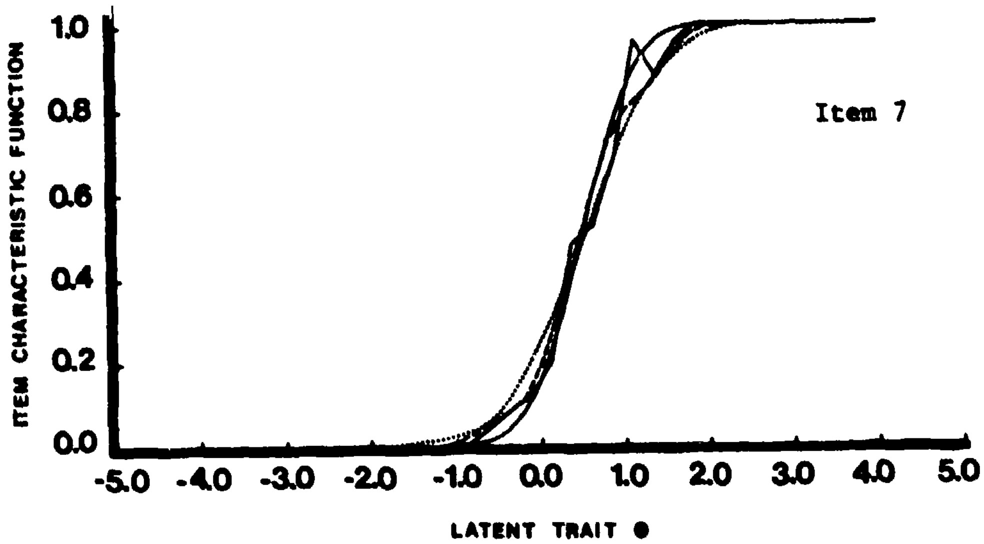




FIGURE 4-3 (Continued)
Subtest 6



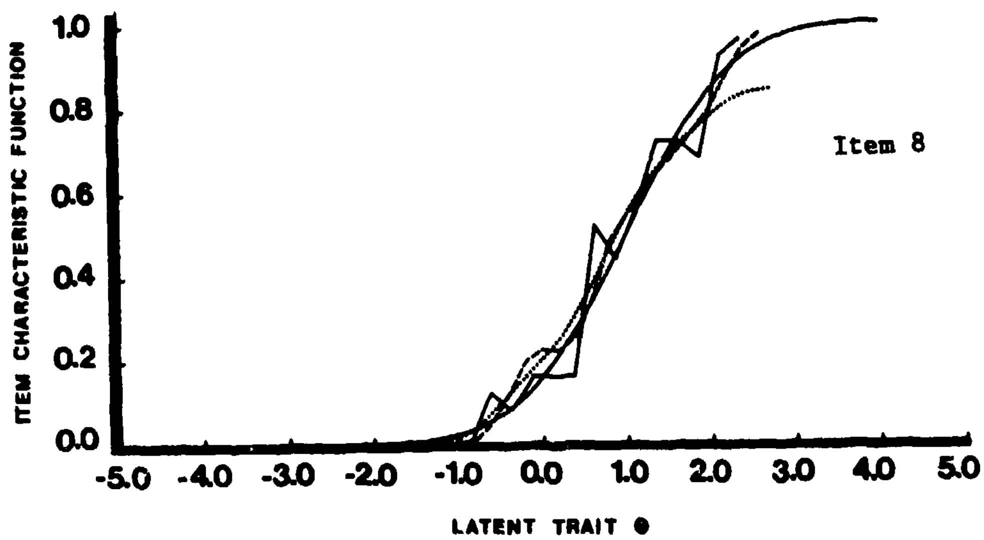




FIGURE 4-3 (Continued)
Subtest 6

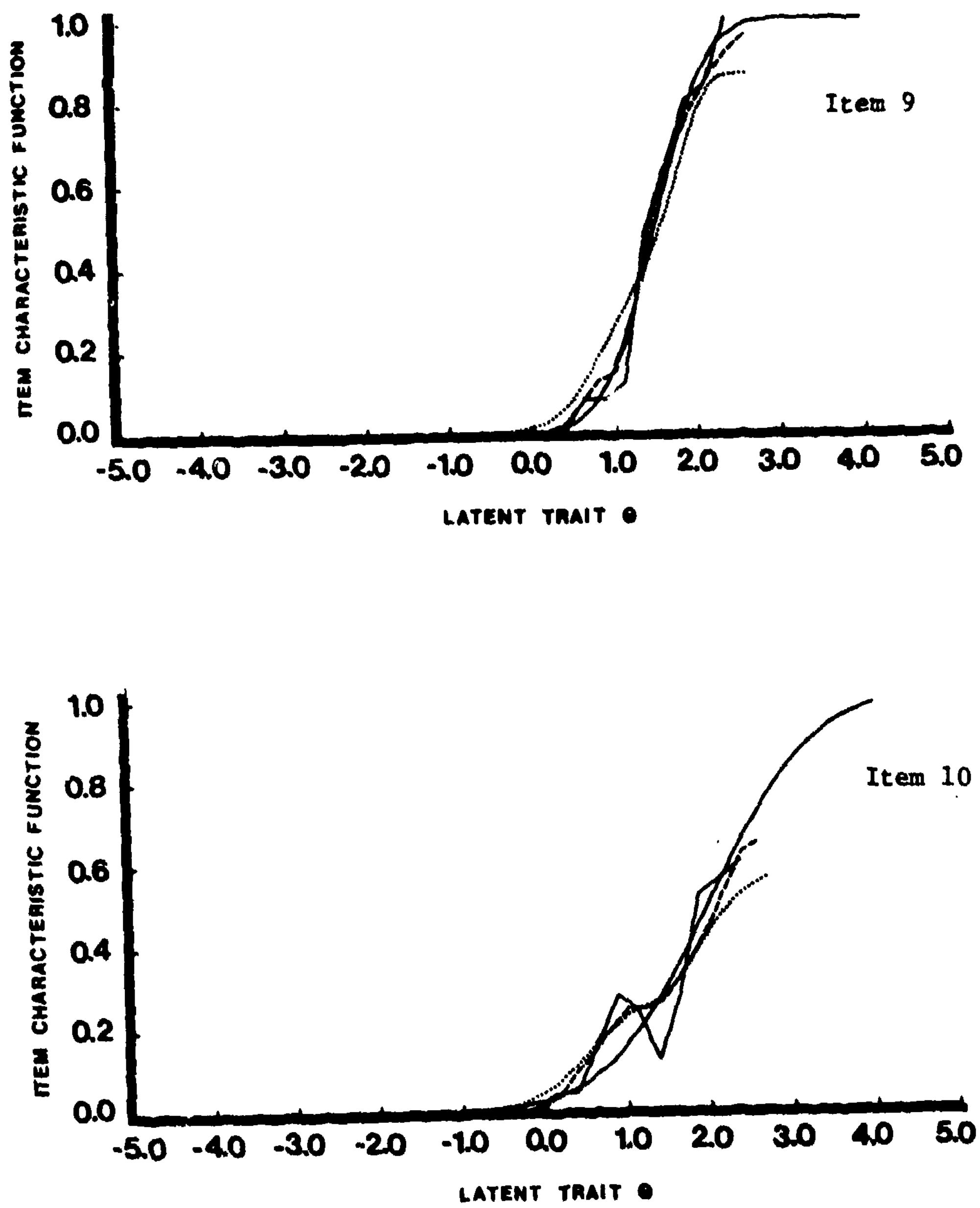



FIGURE 4-3 (Continued)
Subtest 6

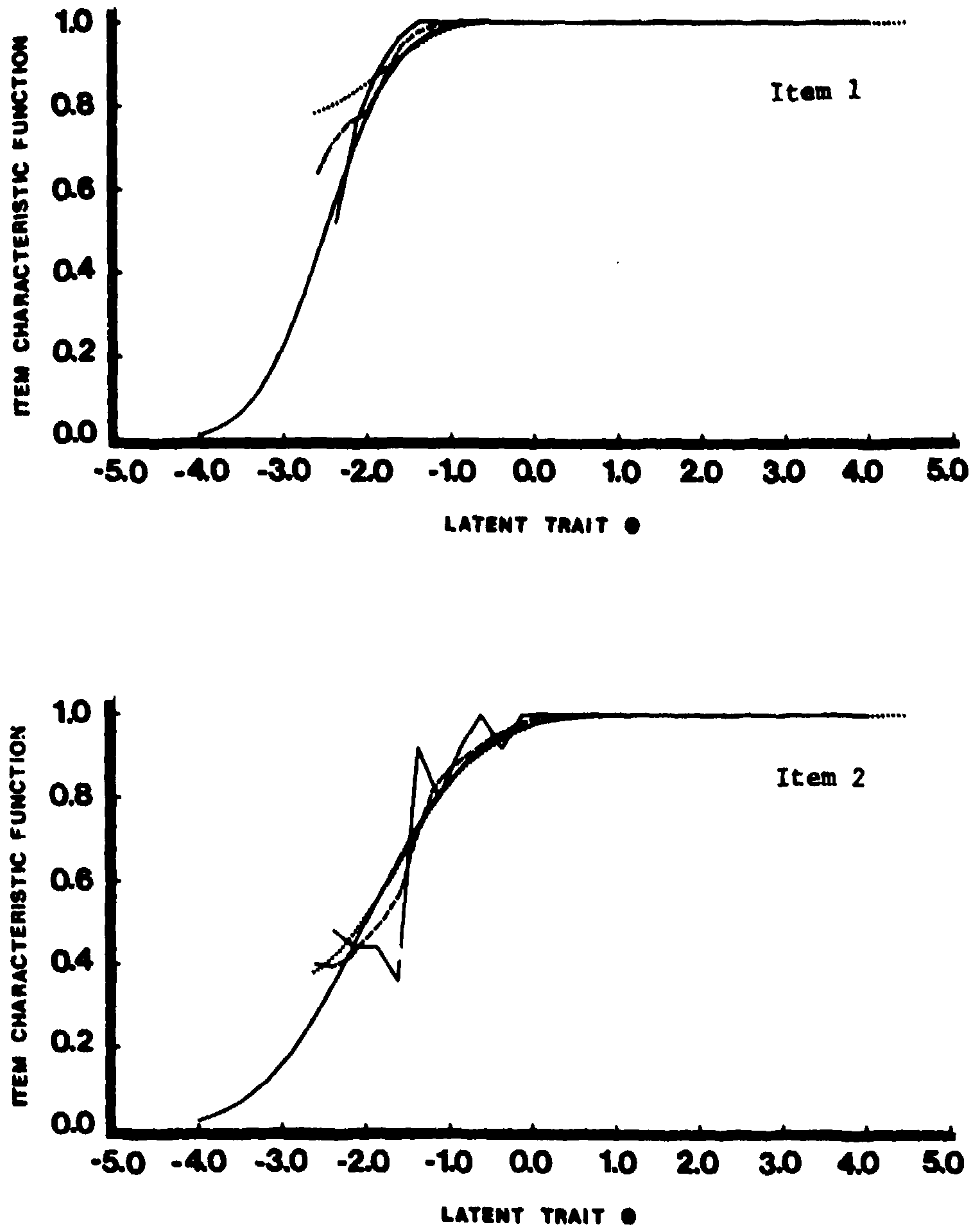
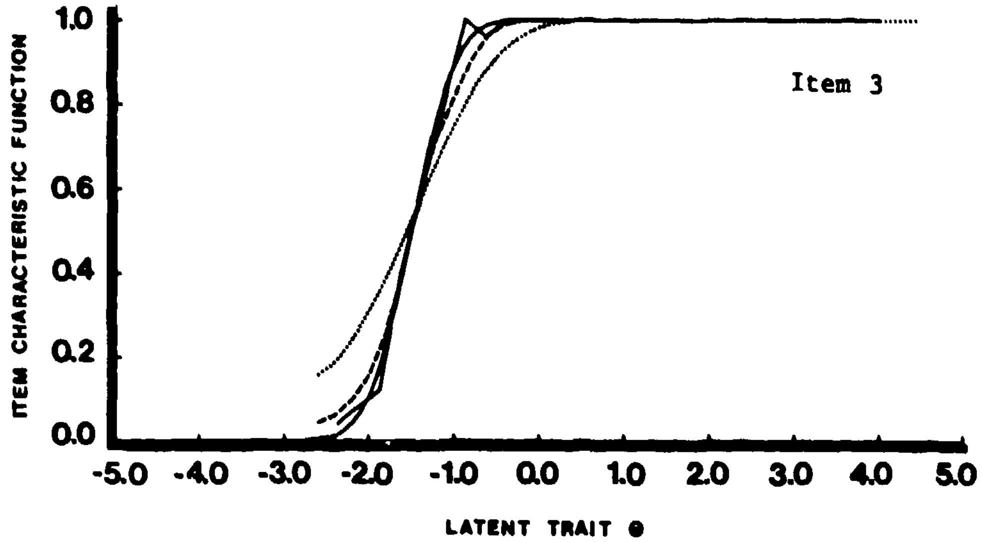



FIGURE 4-4

Estimated Item Characteristic Function Based upon Subtest 7 (Dotted Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method, for Degree 4 Case, in Comparison with the One Based upon the Original Old Test (Dashed Curve), the Theoretical Item Characteristic Function (Smooth Solid Curve) and the Frequency Ratios of Those Who Answered Correctly (Jagged Solid Line).



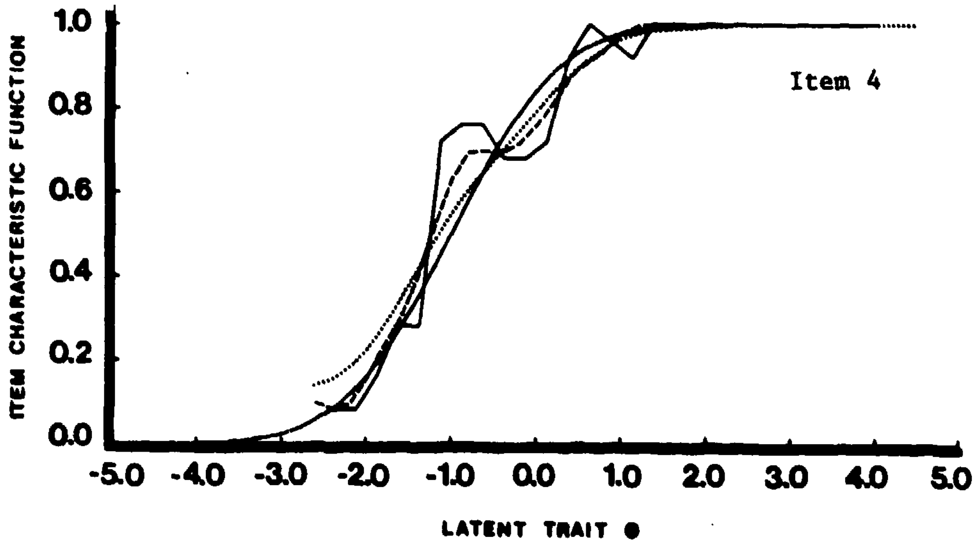
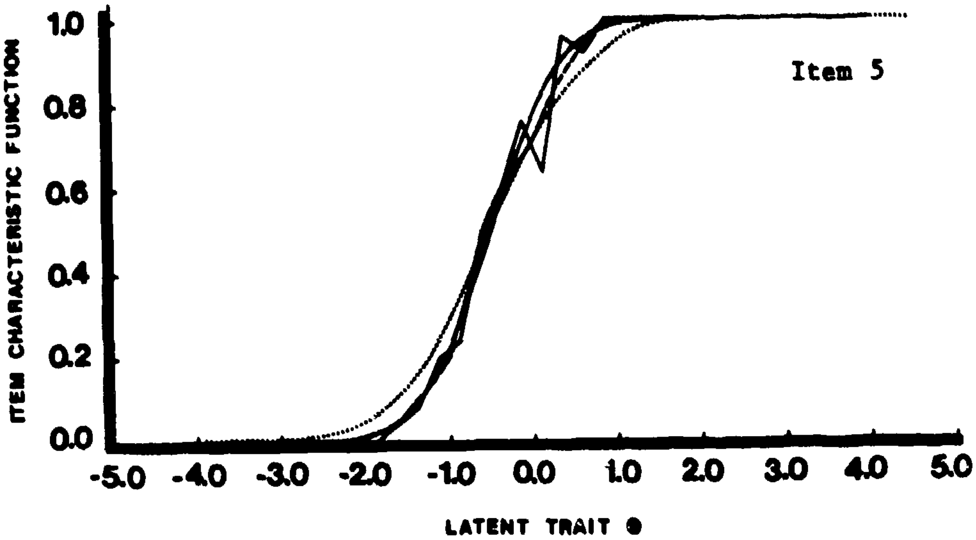




FIGURE 4-4 (Continued)
Subtest 7



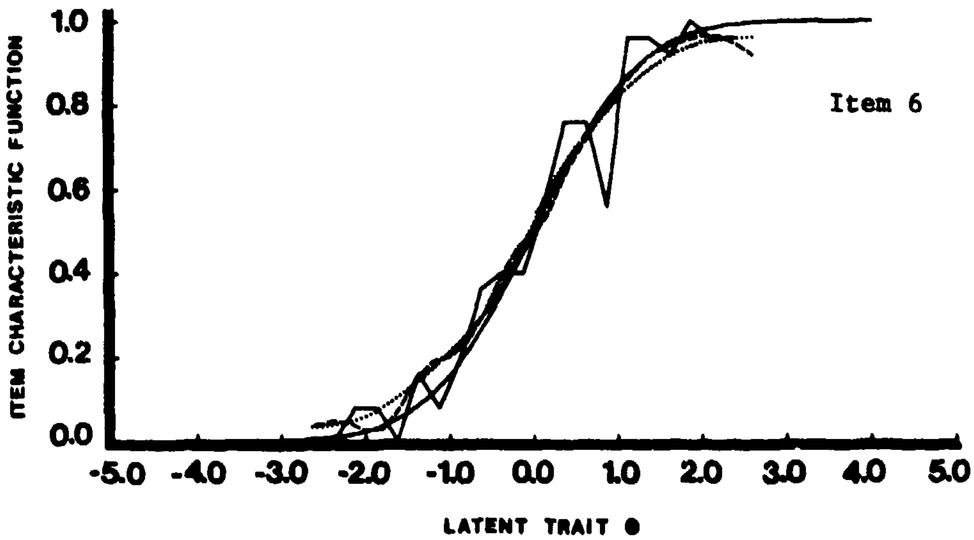
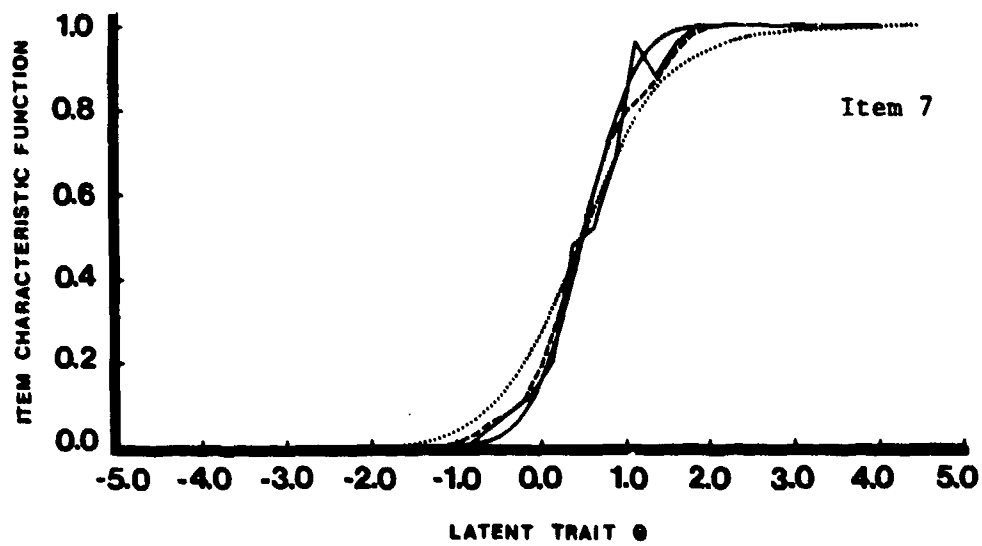




FIGURE 4-4 (Continued)
Subtest 7



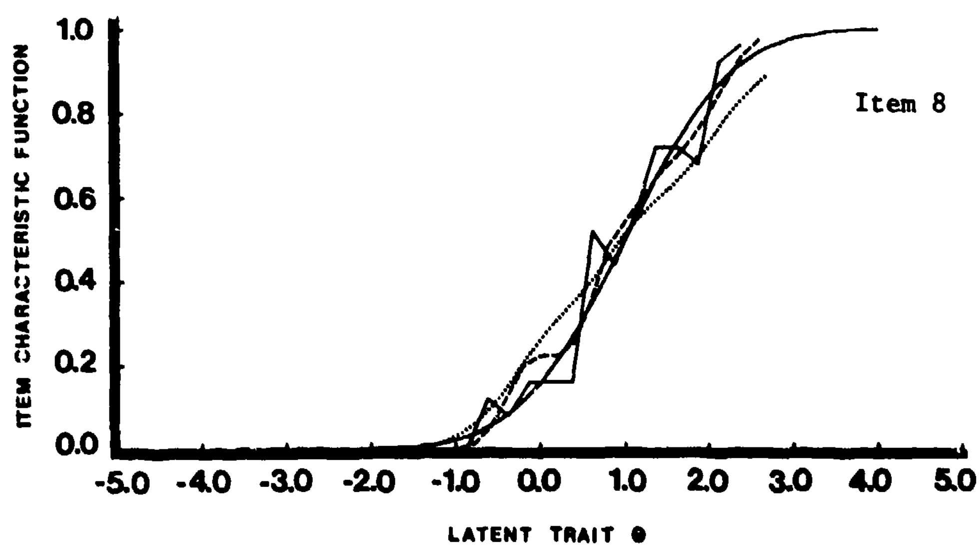
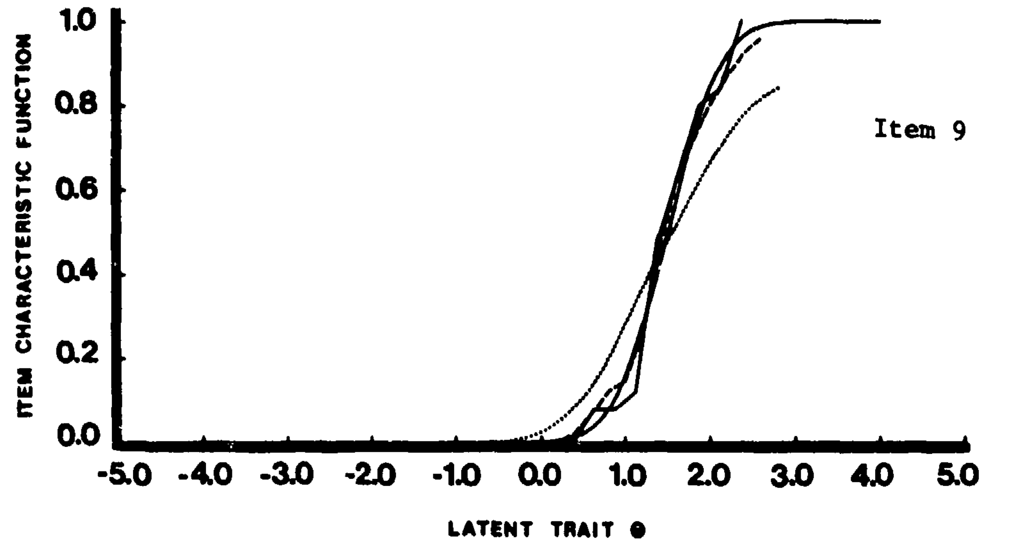




FIGURE 4-4 (Continued)
Subtest 7



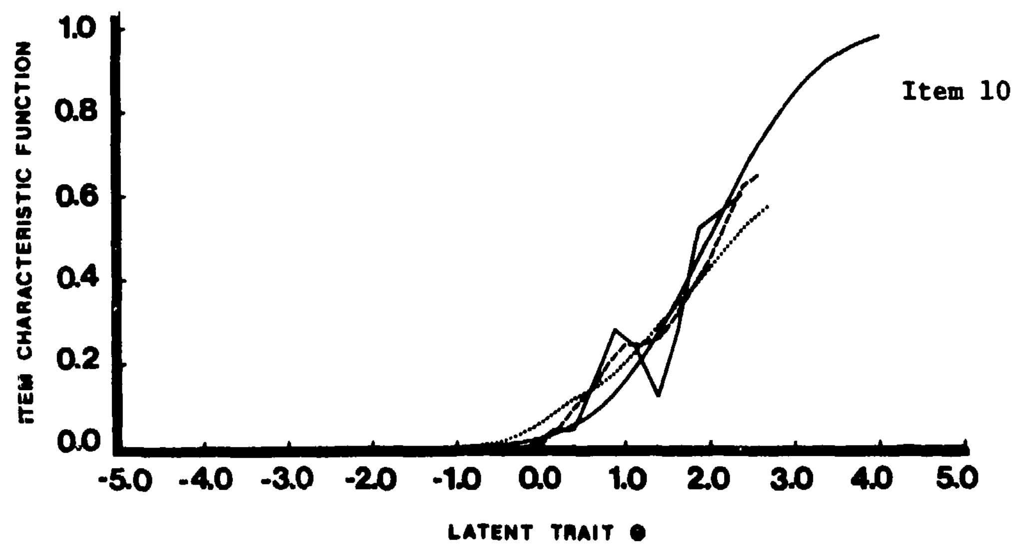
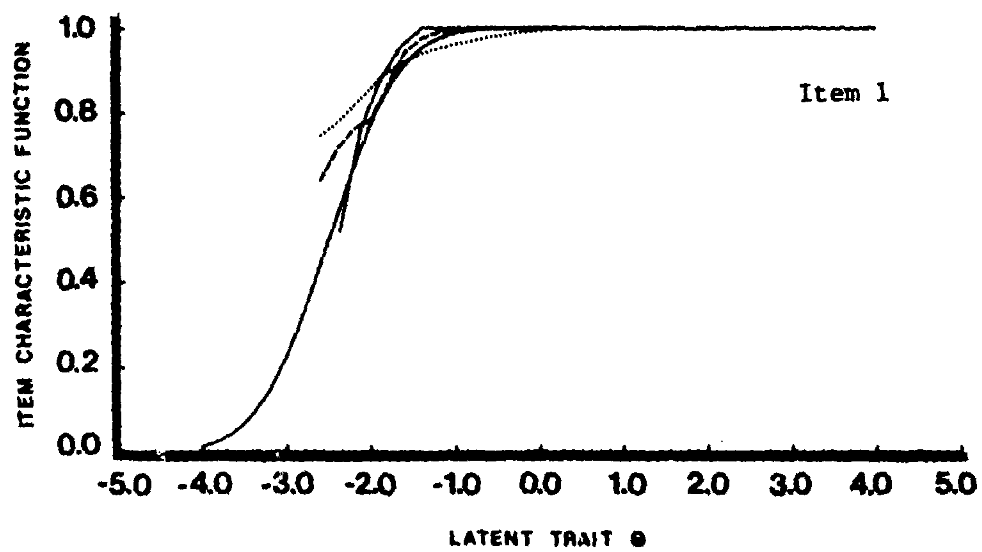




FIGURE 4-4 (Continued)
Subtest 7



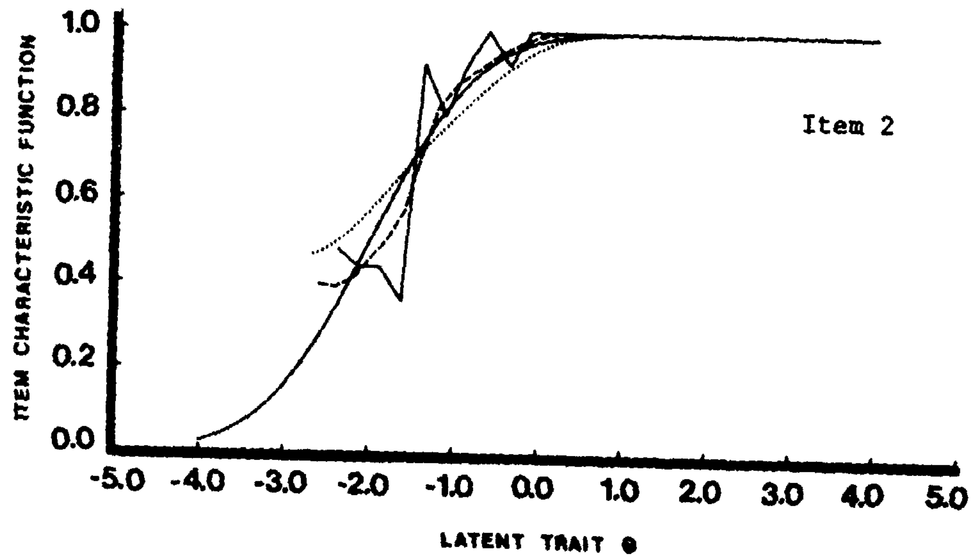
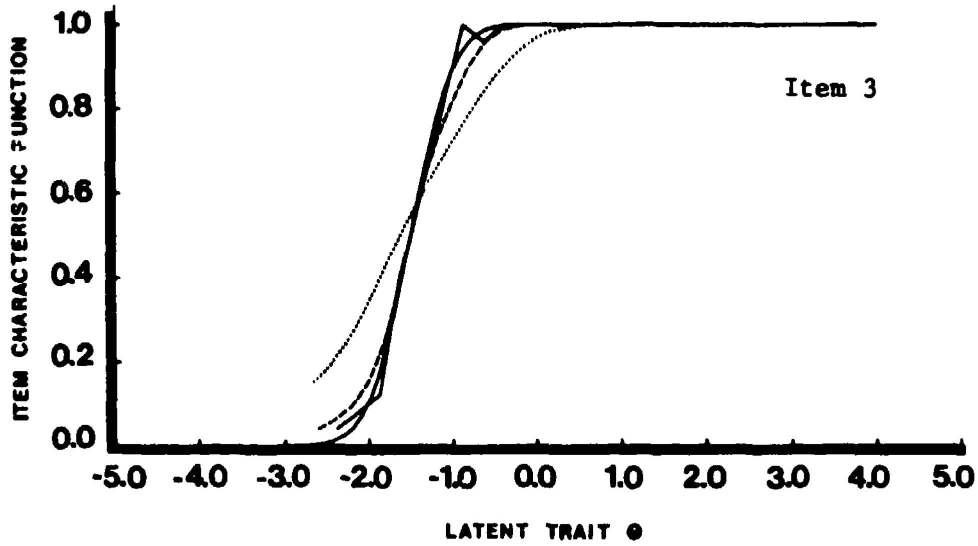




FIGURE 4-5

Estimated Item Characteristic Function Based upon Subtest 8 (Dotted Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method, for Degree 4 Case, in Comparison with the One Based upon the Original Old Test (Dashed Curve), the Theoretical Item Characteristic Function (Smooth Solid Curve) and the Frequency Ratios of Those Who Answered Correctly (Jagged Solid Line).



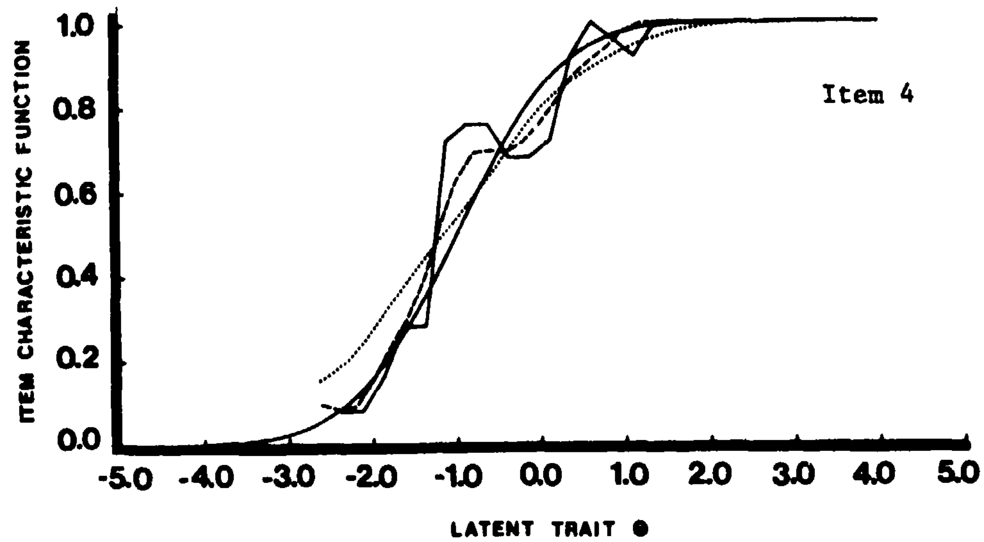
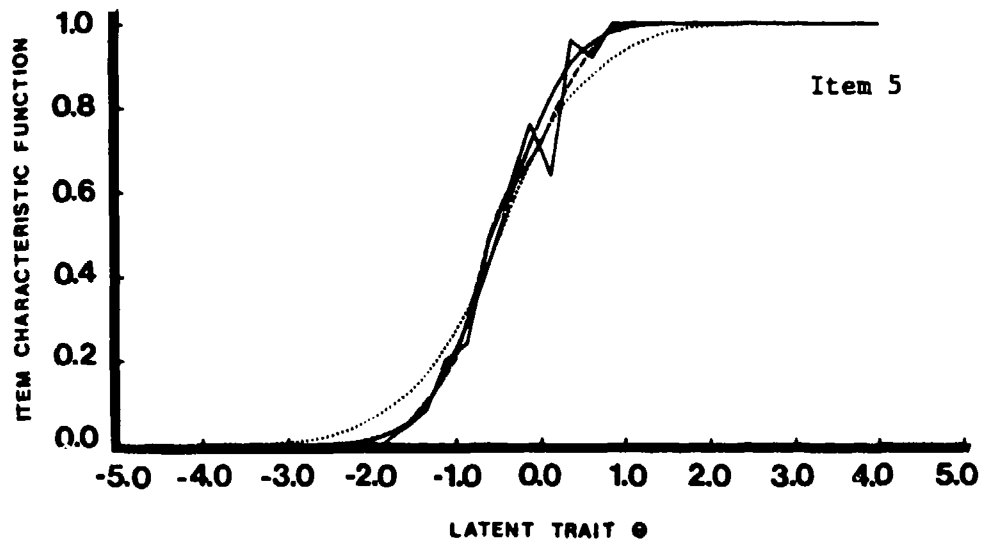




FIGURE 4-5 (Continued)
Subtest 8



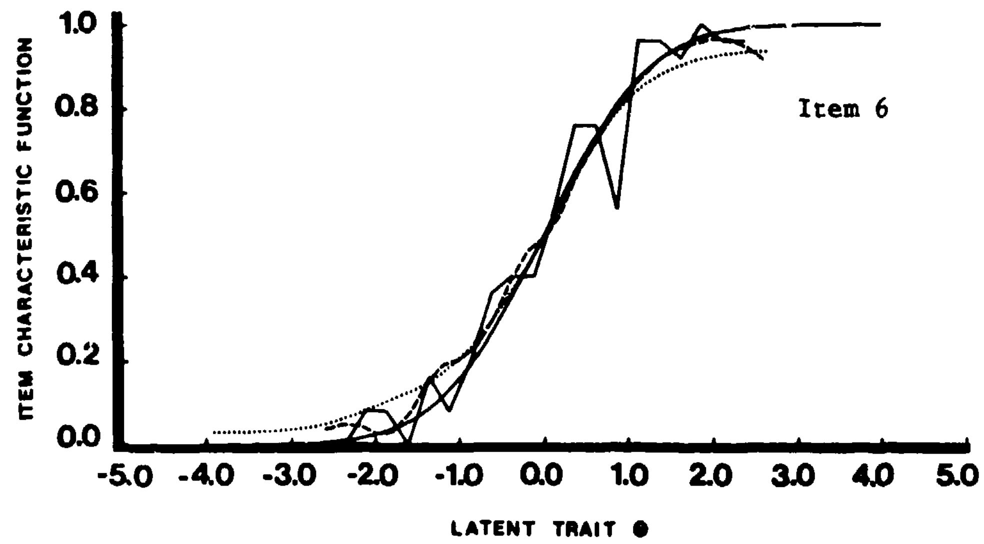
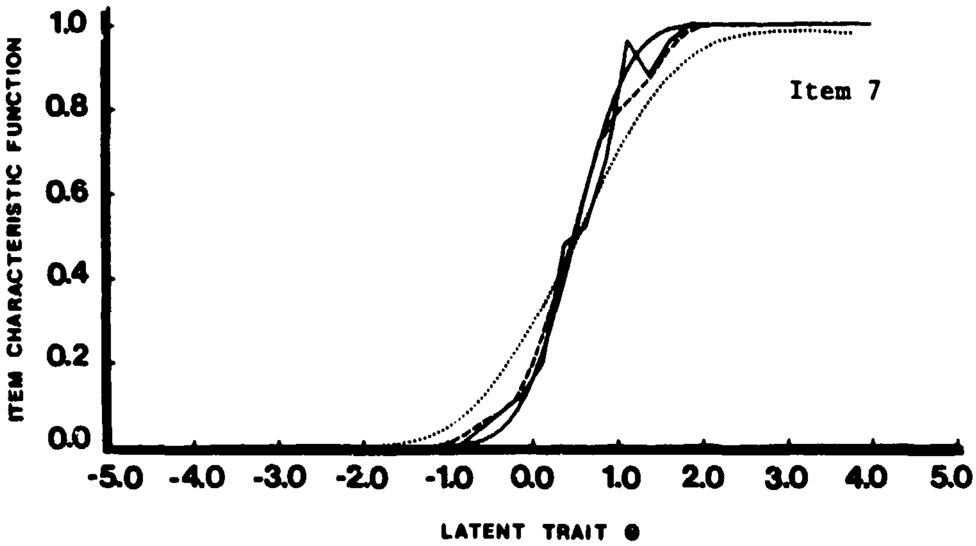




FIGURE 4-5 (Continued)
Subtest 8



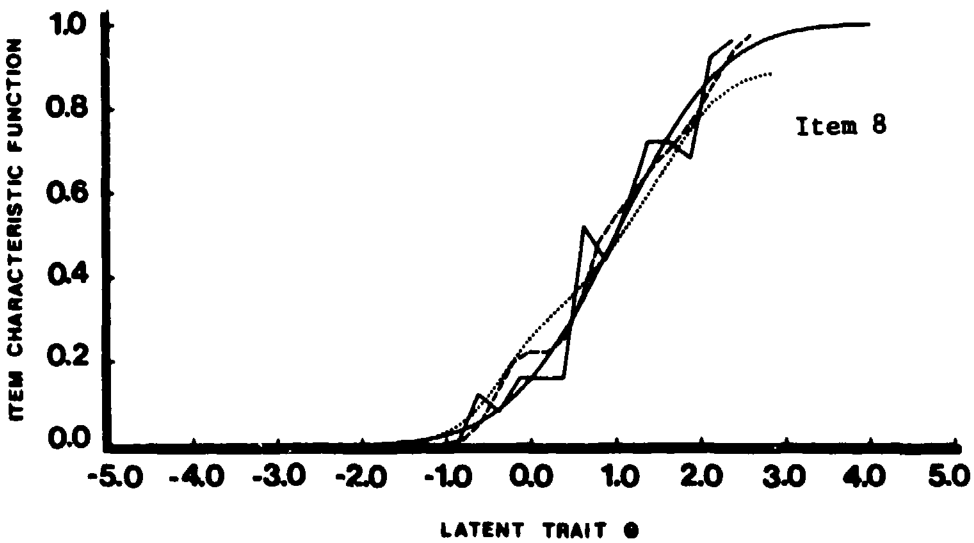
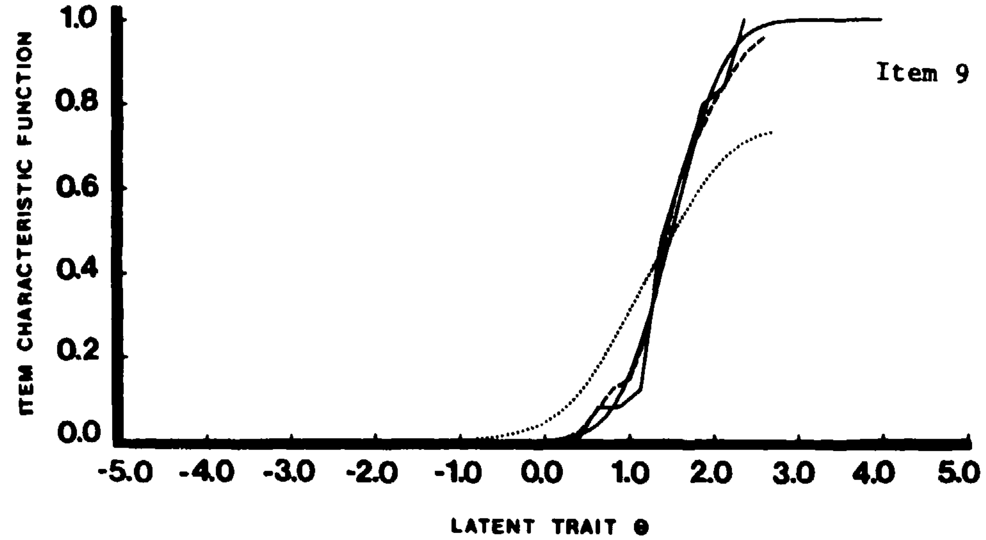




FIGURE 4-5 (Continued)
Subtest 8



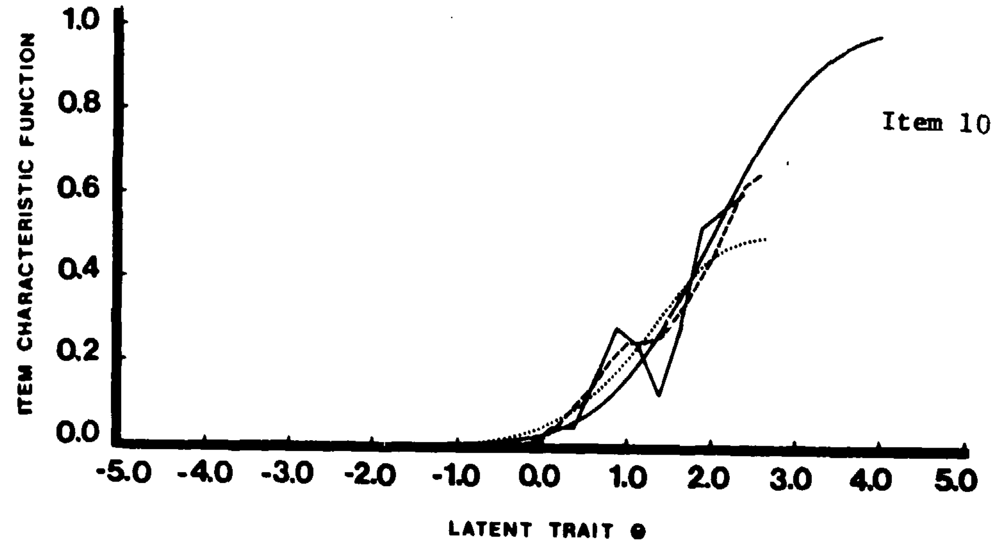
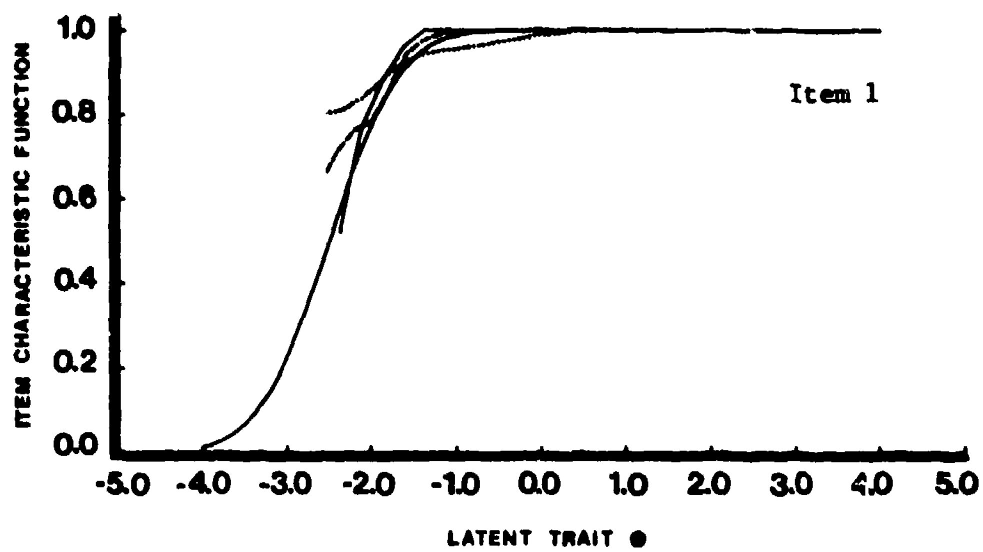




FIGURE 4-5 (Continued)
Subtest 8



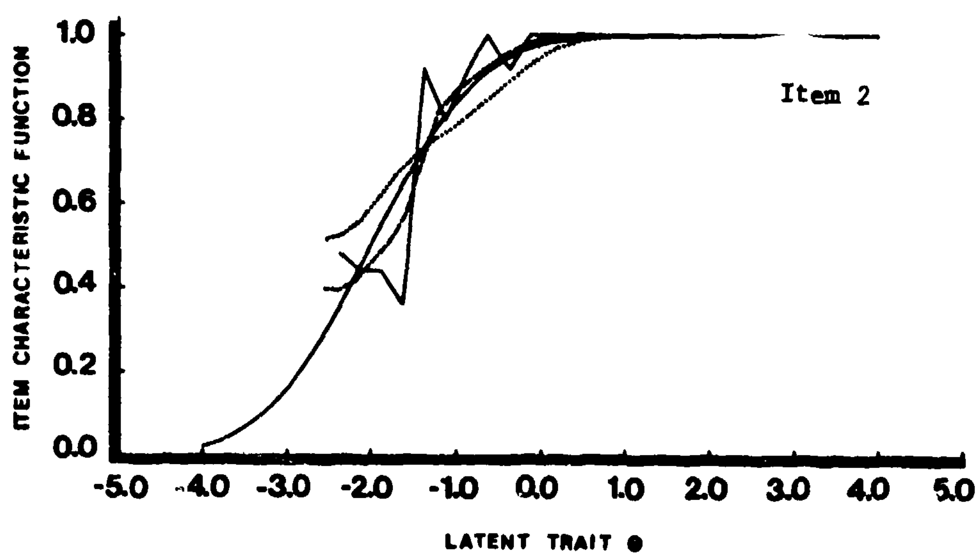
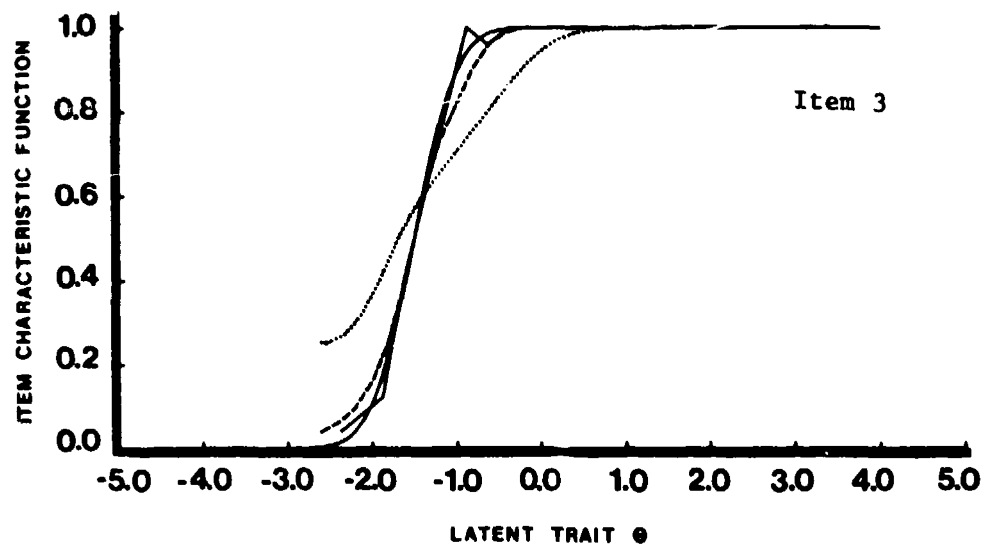




FIGURE 4-6

Estimated Item Characteristic Function Based upon Subtest 9 (Dotted Curve), Obtained by the Simple Sum Procedure of the Conditional P.D.F. Approach and the Normal Approach Method, for Degree 4 Case, in Comparison with the One Based upon the Original Old Test (Dashed Curve), the Theoretical Item Characteristic Function (Smooth Solid Curve) and the Frequency Ratios of Those Who Answered Correctly (Jagged Solid Line).



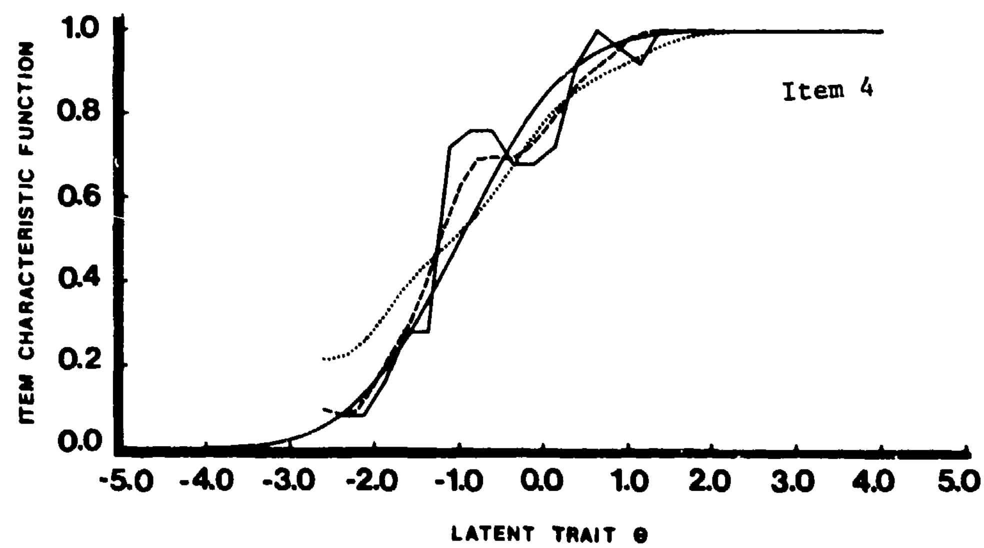
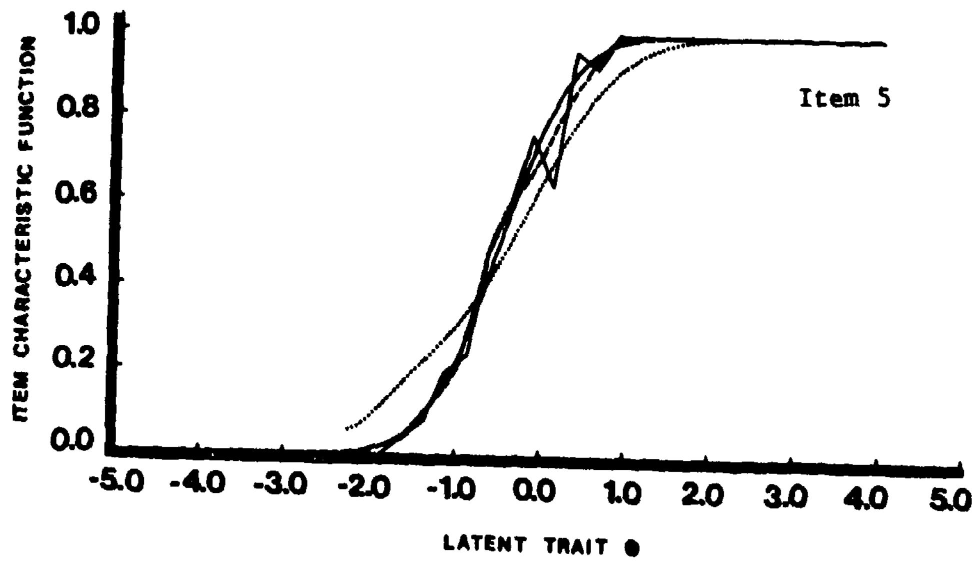




FIGURE 4-6 (Continued)
Subtest 9



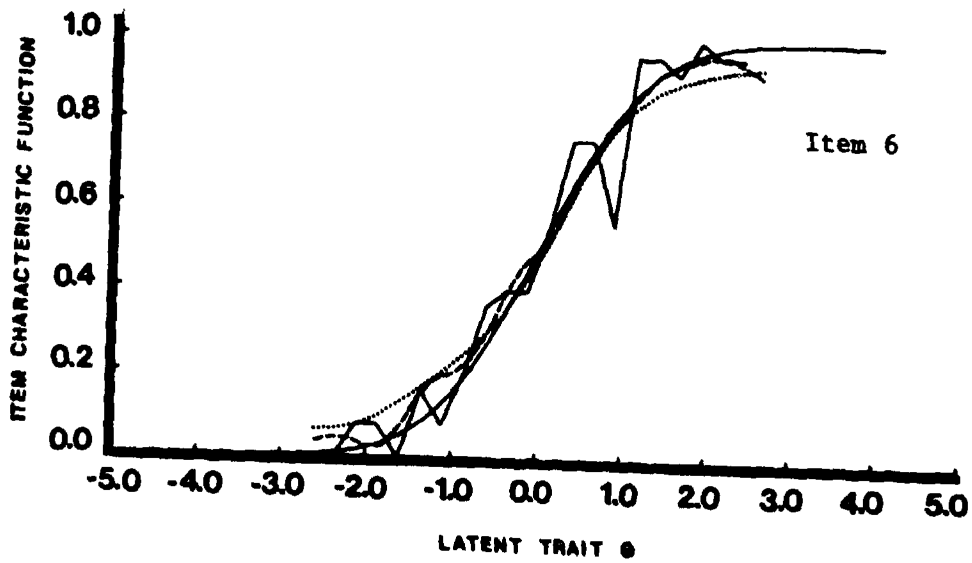
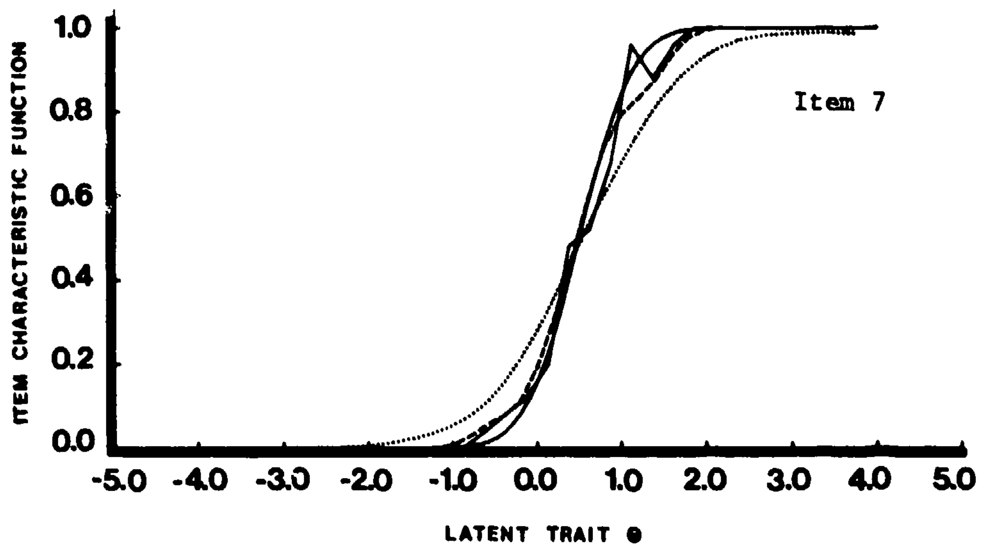




FIGURE 4-6 (Continued)
Subtest 9



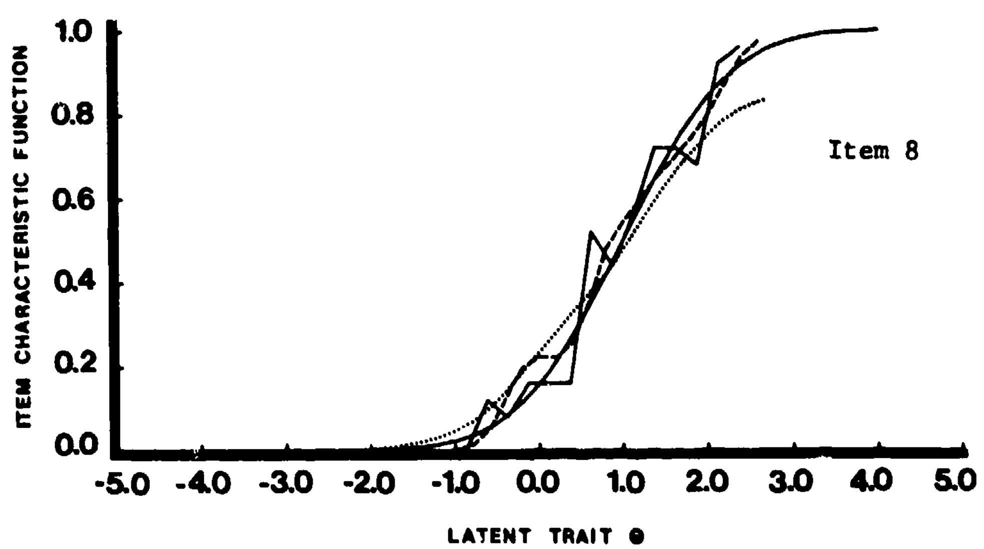
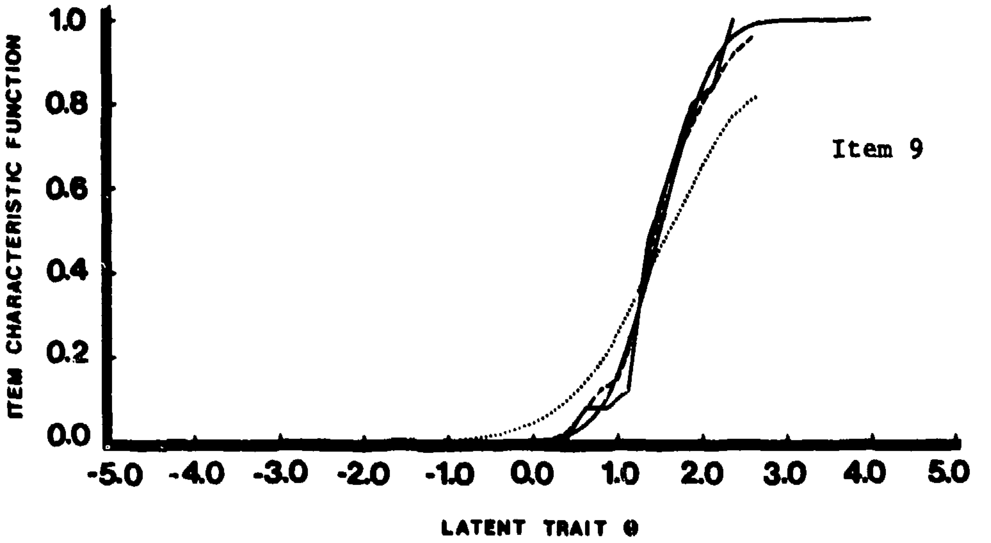




FIGURE 4-6 (Continued)
Subtest 9



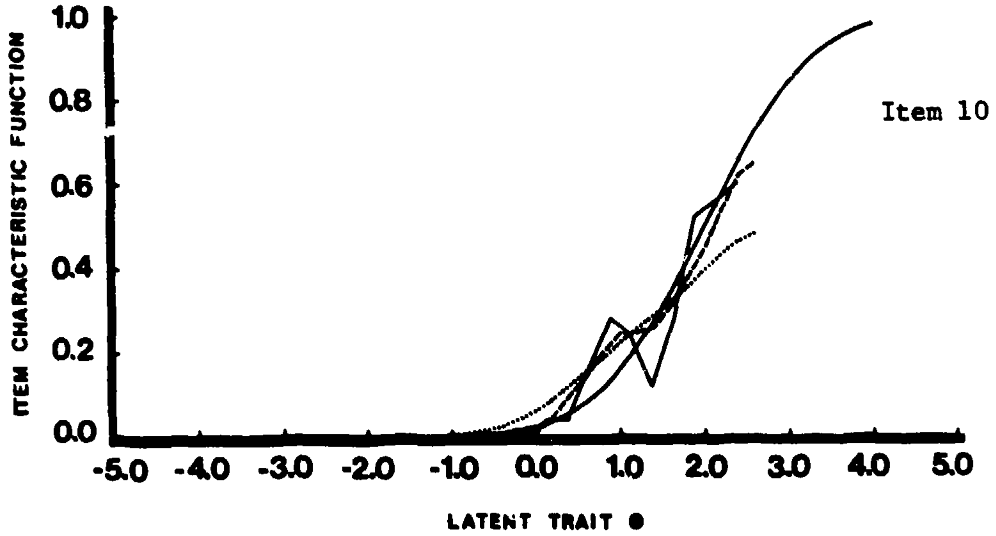




FIGURE 4-6 (Continued)
Subtest 9

Figure 4-1 that, for each binary test item, the estimated item characteristic function is fairly close to the theoretical item characteristic function. In fact, for items 2, 4 and 8 , the approximations are even better than those based upon the original Old Test. We find, however, that for those items having difficulty parameters close to zero, i.e., items 5, 6 and 7 , the approximations are worse. This is expected from the fact that the square root of the test information function of Subtest 4 assumes low values around $\theta = 0.0$ , as we have seen in Figure 2-1 .

## V <u>Linear Regressions of the Maximum Likelihood Estimate</u> $\hat{\tau}_s$ <u>on Ability</u> $\tau_s$

Figure 5-1 presents the scatter diagram of our hypothetical examinees with respect to their ability levels, $\tau_s$ , and their maximum likelihood estimates, $\hat{\tau}_s$ , for each of the six subtests. There are 500 plots in the first five graphs for Subtests 4 through 8, and 498 in the last graph for Subtest 9 (cf. Chapter 3). In each graph of Figure 5-1, also presented is a straight line with 45 degrees from the abscissa, which indicates the asymptotic unbiasedness of the maximum likelihood estimate. We can see in these graphs that for such subtests as Subtests 4, 7, 8 and 9 there are some isolated clusters resultant from small amounts of test information. In other words, there are subgroups of examinees whose response patterns, and consequently maximum likelihood estimates, are identical for such subtests. This tendency becomes more conspicuous as the number of test items in the Old Test decreases.

Table 5-1 presents the coefficients of the sample linear regression, of $\hat{\tau}$ on ability $\tau$ , which is given by $\alpha\tau + \beta$ , for each of the six subtests. This sample linear regression is the best fitted linear function of $\tau$ which makes the sum total of the squared discrepancies of $\hat{\tau}$ minimal. The values of $\alpha$ and $\beta$ turned out to be very close to unity and zero, respectively, for all the six subtests. This is the reason why they are not drawn in Figure 5-1, and if they are, they will be indistinguishable from the 45 degree lines.

Close examination of each of the six scatter diagrams of Figure 5-1 reveals that the sample conditional distribution of $\hat{\tau}$ , given $\tau$ , is not too close to the normal distribution for certain ranges of $\tau$ for Subtests 4, 7, 8 and 9 . The error score, $e_s$ , for each individual examinee $s$ is defined by

$$(5.1) \qquad e_s = [\hat{\tau}_s - \tau_s][I^*(\tau_s)]^{-1/2} \quad .$$

Since for all the subtests $[I^*(\tau)]^{1/2} = 3.5$ , we used this constant in

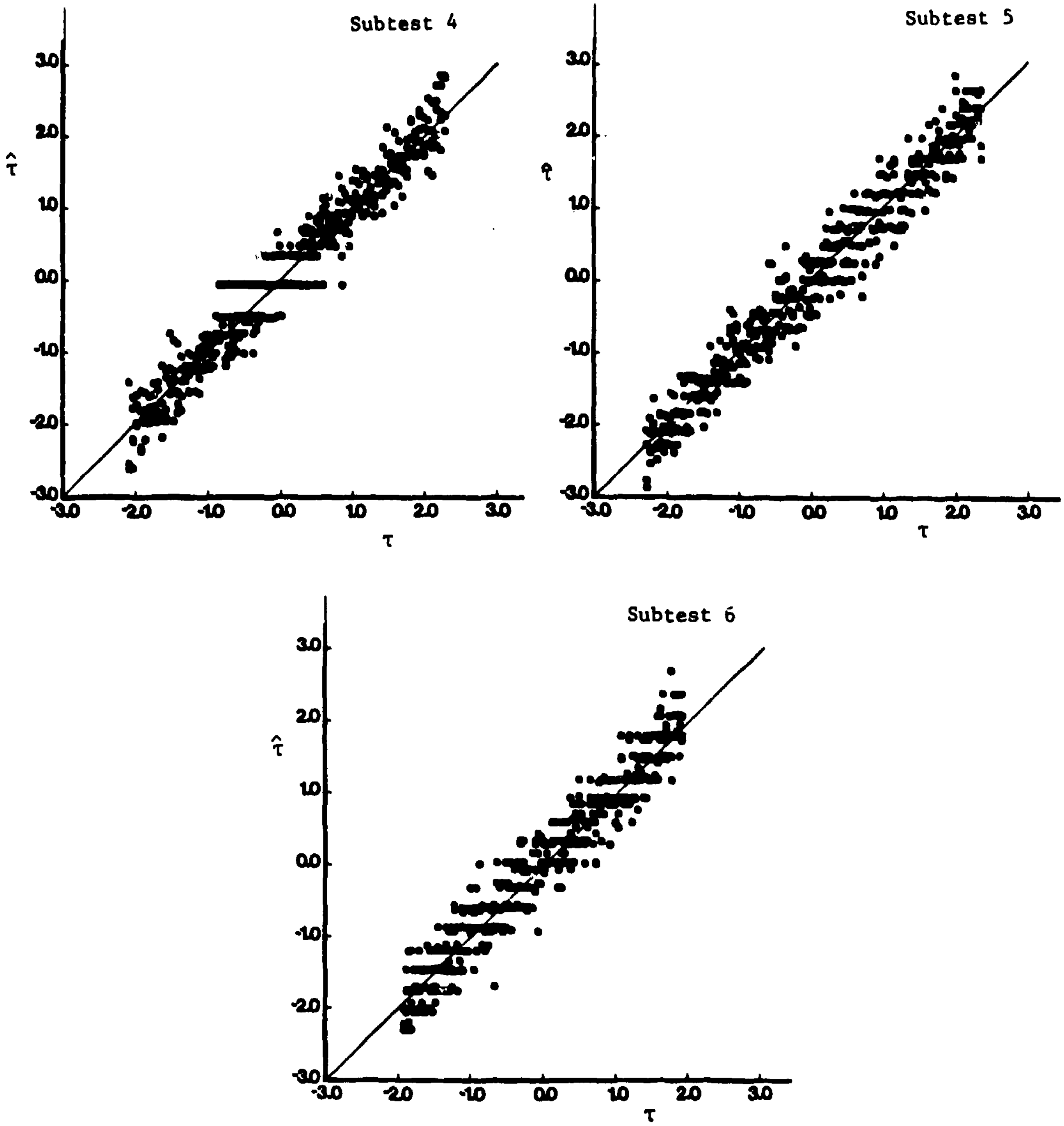



FIGURE 5-1

Scatter Diagram of $\hat{\tau}$ Plotted Against $\tau$ for Our Hypothetical Examinees, Which Is Based upon Each of the Six Subtests.

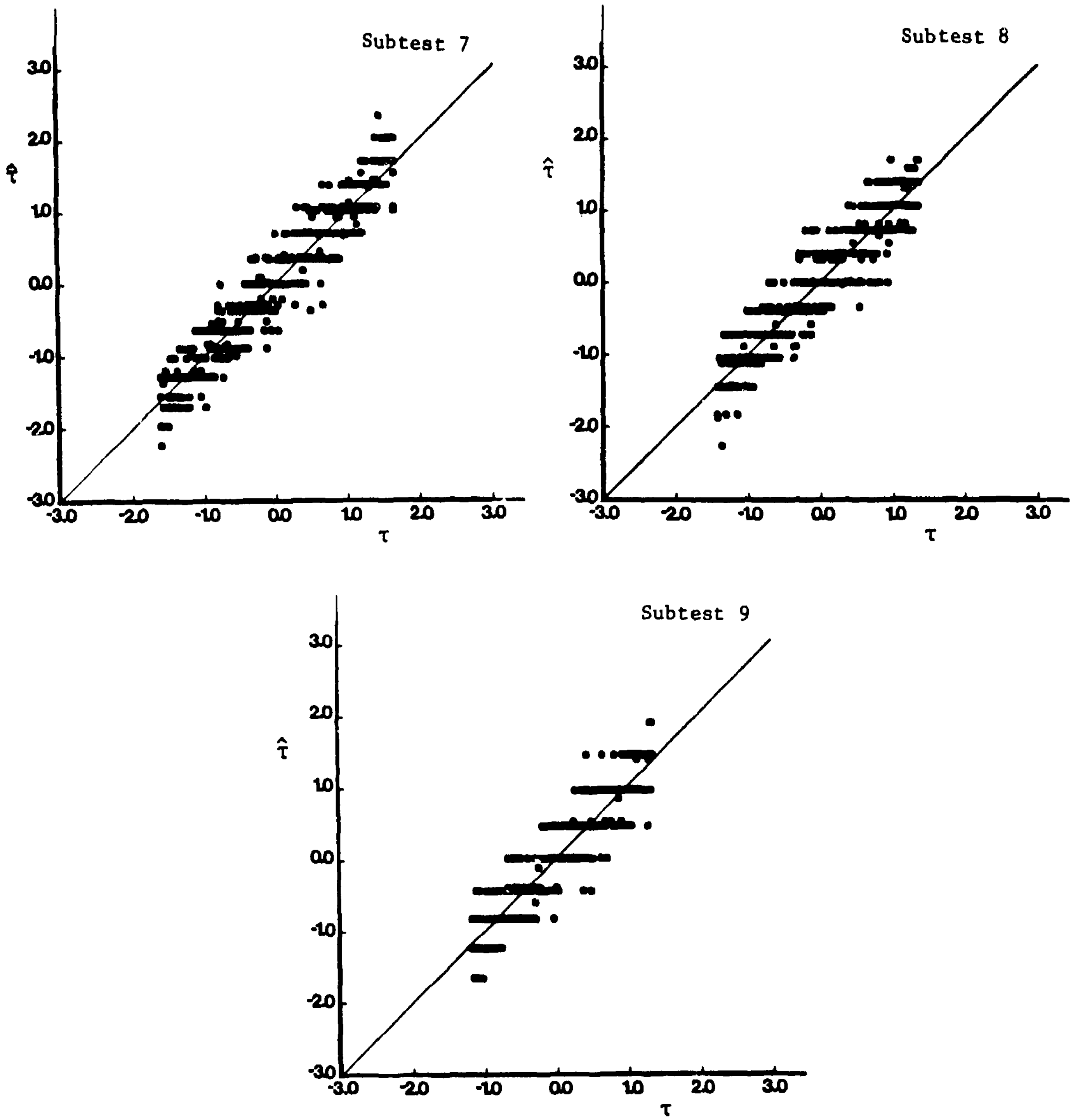



FIGURE 5-1 (Continued)

TABLE 5-1

Coefficients of the Sample Linear Regression of $\hat{\tau}$ on $\tau$ , $\alpha\tau + \beta$ , for Our Hypothetical Examinees, for Each of the Six Subtests.

| Subtest | $\alpha$ | $\beta$ |
|---|---|---|
| 4 | 1.00266 | 0.00393 |
| 5 | 1.01779 | -0.00650 |
| 6 | 1.01132 | -0.00030 |
| 7 | 1.01634 | -0.00251 |
| 8 | 0.99995 | -0.00865 |
| 9 | 1.00918 | 0.01323 |

(5.1) instead of actual values of $[I^*(\tau_s)]^{1/2}$. If the conditional distribution of $\hat{\tau}$, given $\tau$, is approximately normal, with the true ability $\tau$ and the reciprocal of the square root of the test information function as the two parameters, then the error score $e_s$ will distribute, approximately, normally with zero and unity as its two parameters.

The frequency distribution of the 500 error scores for each of Subtests 4 through 8, and the 498 error scores for Subtest 9, was obtained using the category width of 0.2 . Figure 5-2 presents these results in the form of histograms, together with the standard normal density function, which is drawn by a dotted curve in each of the six graphs. The chi-square test for the goodness of fit was made for each histogram against the standard normal density function by combining all the subintervals below $e = -2.8$ into one category, and all subintervals above $e = 2.8$ into another, to give thirty categories in total and make the number of degrees of freedom in each chi-square test 29 . As it turned out, for Subtest 4 we obtained $\chi^2 = 32.812$ which provides us with $0.25 < p < 0.50$ ; for Subtest 5, $\chi^2 = 28.269$ , $0.50 < p < 0.75$ ; for Subtest 6, $\chi^2 = 18.878$ , $0.90 < p < 0.95$ ; for Subtest 7, $\chi^2 = 32.574$ , $0.25 < p < 0.50$ ; for Subtest 8, $\chi^2 = 33.695$ , $0.25 < p < 0.50$ ; and for Subtest 9, $\chi^2 = 26.166$ , which provides $0.50 < p < 0.75$ .

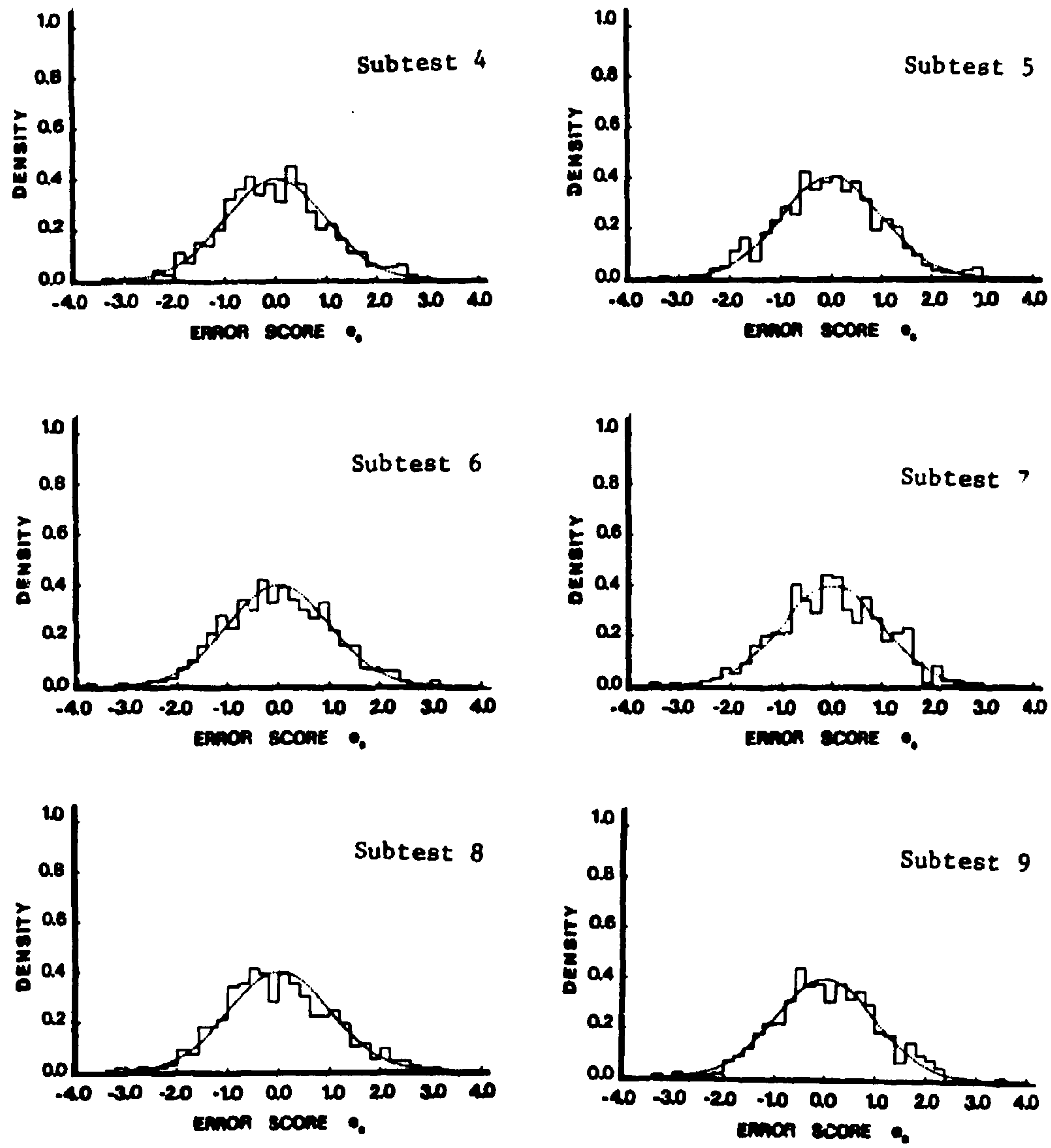



FIGURE 5-2

Frequency Distribution of the Error Score $e_s$ (Solid Line) of Our Hypothetical Examinees, in Comparison with the Standard Normal Density Function (Dotted Curve).

## VI Discussion and Conclusions

Following the previous studies (Samejima, RR-80-4, RR-81-2), we have further investigated the effects of reducing the number of test items in our Old Test on the accuracy of estimation of the item characteristic functions of the ten binary test items, using the same combination of a method and an approach, i.e., Simple Sum Procedure of the Conditional P.D.F. Approach combined with the Normal Approach Method. As it turned out, we were able to successfully estimate the item characteristic functions with as few as eleven test items in our Old Test, i.e., Subtest 6. This is a substantial decrease from the thirty-five items of the original Old Test. It appears that an appropriate strategy is to select test items for the Old Test whose difficulty parameters are fairly evenly spaced over the interval of ability of our interest. Once again, the method of moments proved useful in the present study.

When fewer than eleven test items were used as the Old Test, i.e. Subtests 7, 8 and 9, the estimation of the operating characteristics was not so successful. We may conclude, therefore, the border line is between Subtest 6 and Subtest 7 to separate those which are acceptable as our Old Test and those which are not.

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# APPENDIX

TABLE A-1

Coefficients of the Polynomials of Degrees 3 Through 6 Obtained by the Method of Moments Using the Interval of $\theta$ , (-4.0, 4.0) , to Approximate the Square Root of the Test Information Function for Each of the Six Subtests.

| k | | Coefficient $\alpha_k$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | Subtest 4 | Subtest 5 | Subtest 6 | Subtest 7 | Subtest 8 | Subtest 9 |
| 0 | DGR. 3 | 0.31206938D 01 | 0.34719207D 01 | 0.28426908D 01 | 0.23785492D 01 | 0.20168133D 01 | 0.18599054D 01 |
| 1 | | 0.81764656D-01 | 0.15939219D-02 | 0.32597374D-01 | 0.80980570D-02 | -0.52322534D-01 | 0.11859674D 00 |
| 2 | | -0.15778598D-01 | -0.88010524D-01 | -0.47885080D-01 | -0.20638913D-01 | -0.75901590D-02 | -0.28337192D-01 |
| 3 | | -0.56994549D-02 | 0.16471406D-03 | -0.24652422D-02 | 0.21448331D-03 | 0.16593340D-02 | -0.81065900D-02 |
| 0 | DGR. 4 | 0.23414354D 01 | 0.32871441D 01 | 0.27476093D 01 | 0.22944801D 01 | 0.19492549D 01 | 0.17669089D 01 |
| 1 | | 0.81764656D-01 | 0.15939219D-02 | 0.32597374D-01 | 0.80980570D-02 | -0.52322534D-01 | 0.11859674D 00 |
| 2 | | 0.47125790D 00 | 0.27474848D-01 | 0.11540869D-01 | 0.31904272D-01 | 0.34633864D-01 | 0.29785621D-01 |
| 3 | | -0.56994549D-02 | 0.16471406D-03 | -0.24652422D-02 | 0.21448331D-03 | 0.16593340D-02 | -0.81065900D-02 |
| 4 | | -0.35513078D-01 | -0.84208084D-02 | -0.43331421D-02 | -0.38312739D-02 | -0.30788350D-02 | -0.42381218D-02 |
| 0 | DGR. 5 | 0.23414354D 01 | 0.32871441D 01 | 0.27476093D 01 | 0.22944801D 01 | 0.19492549D 01 | 0.17669089D 01 |
| 1 | | 0.19905059D 00 | 0.10040261D 00 | 0.10420599D-01 | 0.75610393D-01 | -0.92811121D-03 | 0.17007735D 00 |
| 2 | | 0.47125790D 00 | 0.27474848D-01 | 0.11540869D-01 | 0.31904272D-01 | 0.34633864D-01 | 0.29785621D-01 |
| 3 | | -0.39907852D-01 | -0.28654487D-01 | 0.40029839D-02 | -0.19476615D-01 | -0.13330706D-01 | -0.23121767D-01 |
| 4 | | -0.35513078D-01 | -0.84208084D-02 | -0.43331421D-02 | -0.38312739D-02 | -0.30788350D-02 | -0.42381218D-02 |
| 5 | | 0.19242223D-02 | 0.16210800D-02 | -0.36383772D-03 | 0.11076243D-02 | 0.84318976D-03 | 0.84460370D-03 |
| 0 | DGR. 6 | 0.20653741D 01 | 0.33028985D 01 | 0.28244081D 01 | 0.23698838D 01 | 0.20794533D 01 | 0.18031475D 01 |
| 1 | | 0.19905059D 00 | 0.10040261D 00 | 0.10420599D-01 | 0.75610393D-01 | -0.92811121D-03 | 0.17007735D 00 |
| 2 | | 0.83359597D 00 | 0.68038274D-02 | -0.89251467D-01 | -0.67057488D-01 | -0.13624656D 00 | -0.17773403D-01 |
| 3 | | -0.39907852D-01 | -0.28654487D-01 | 0.40029839D-02 | -0.19476615D-01 | -0.13330706D-01 | -0.23121767D-01 |
| 4 | | -0.10345640D 00 | -0.45492312D-02 | 0.14561530D-01 | 0.14720463D-01 | 0.28958024D-01 | 0.46765970D-02 |
| 5 | | 0.19242223D-02 | 0.16210800D-02 | -0.36383772D-03 | 0.11076243D-02 | 0.84318976D-03 | 0.84460370D-03 |
| 6 | | 0.31140746D-02 | -0.17744225D-03 | -0.86600132D-03 | -0.85028384D-03 | -0.14683524D-02 | -0.40858826D-03 |

TABLE A-2

Coefficients of the Polynomial of Degrees 3 Through 7 Obtained by the Method of Moments Using the Interval of $\theta$, (-3.0, 3.0), to Approximate the Square Root of the Test Information Function of Subtest 4.

| | | Coefficient |
|---|---|---|
| DGR. 3 | 0 | 2.69165 |
| | 1 | 0.20035 |
| | 2 | 0.16279 |
| | 3 | -0.02765 |
| DGR. 4 | 0 | 2.02871 |
| | 1 | 0.20035 |
| | 2 | 0.89938 |
| | 3 | -0.02765 |
| | 4 | -0.09549 |
| DGR. 5 | 0 | 2.02871 |
| | 1 | 0.37170 |
| | 2 | 0.89938 |
| | 3 | -0.11650 |
| | 4 | -0.09549 |
| | 5 | 0.00888 |
| DGR. 6 | 0 | 1.97643 |
| | 1 | 0.37170 |
| | 2 | 1.02139 |
| | 3 | -0.11650 |
| | 4 | -0.13617 |
| | 5 | 0.00888 |
| | 6 | 0.00332 |
| DGR. 7 | 0 | 1.97643 |
| | 1 | 0.53506 |
| | 2 | 1.02139 |
| | 3 | -0.27986 |
| | 4 | -0.13617 |
| | 5 | 0.04882 |
| | 6 | 0.00332 |
| | 7 | -0.00275 |

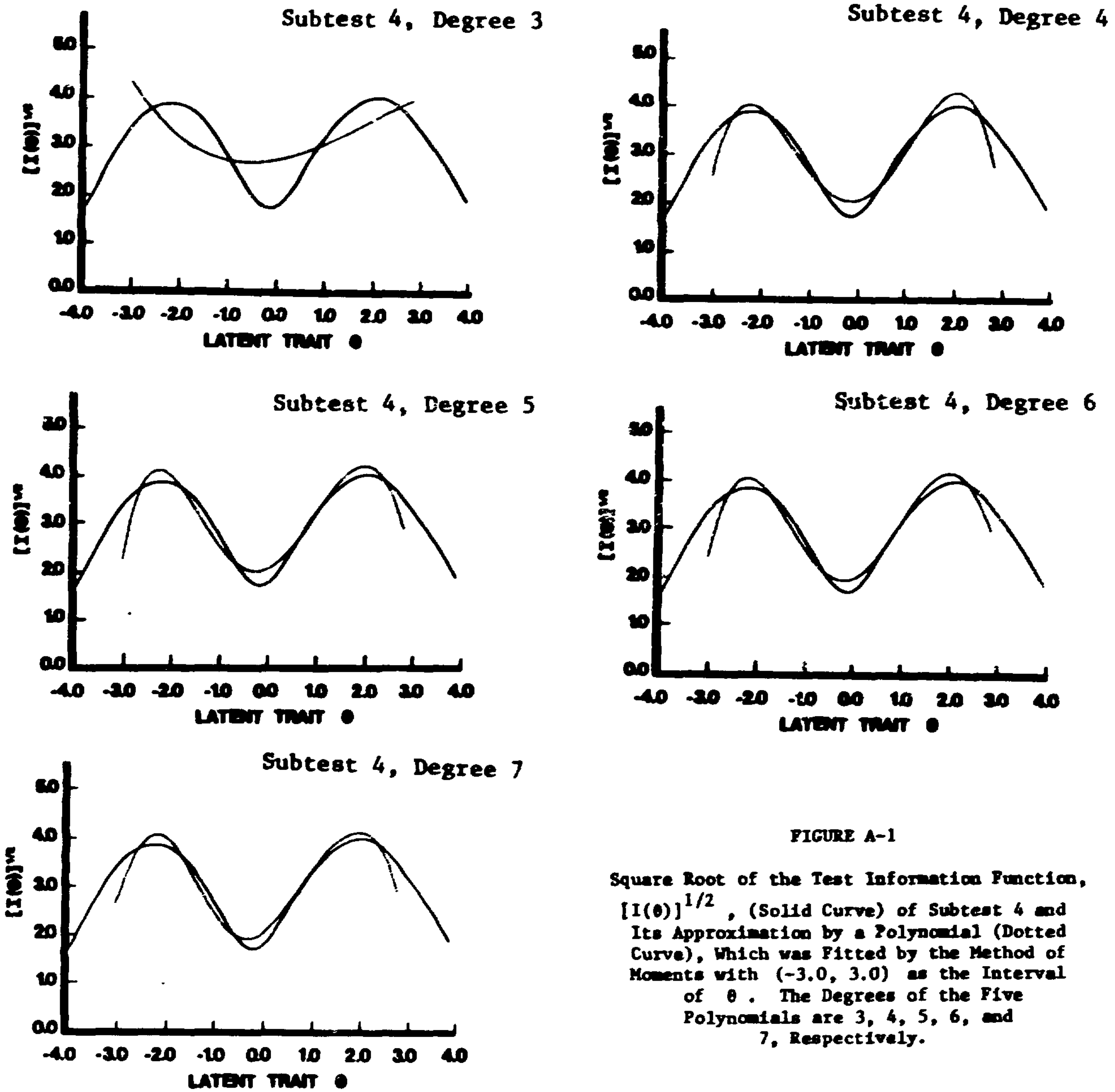


FIGURE A-1

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Curve) of Subtest 4 and Its Approximation by a Polynomial (Dotted Curve), Which was Fitted by the Method of Moments with (-3.0, 3.0) as the Interval of $\theta$. The Degrees of the Five Polynomials are 3, 4, 5, 6, and 7, Respectively.

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